Maximizing System Capacity in a Sub-Symbol based DS-CDMA Multiuser Detector

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Abstract- **A sub-symbol based multiuser detector for DS-CDMA systems was proposed in [1]. This paper addresses the optimization of such detectors to maximize the system capacity. It is shown that the maximum number of simultaneous users supported in a DS-CDMA system using sub-symbol scheme can be increased by the proper selection of certain thresholds delineated in [1], and by imposing moderate synchronization requirements on the transmitters. It is also shown that, in a moderately synchronized environment, performing decorrelation with only one sub-symbol is enough for accommodating a fairly large number of users, if the system signal to noise ratio (SNR) is medium to high. It has been verified through simulations that the maximum number of simultaneous users depends on the degree of synchronism and is approximately given by the number of chips in the longest sub-symbol.**

I. Introduction

CDMA is the preferred air interface technology for third generation wireless systems that are currently undergoing the standardization process. It has been shown previously that the capacity of CDMA systems is limited by multiple access interference (MAI) and not by thermal noise [2]. Currently deployed commercial CDMA systems regard MAI as an additive noise and perform detection of each user independently. This limits the maximum number of users that can be simultaneously supported due to the deterioration of the bit error rate (BER). A number of researchers have addressed multiuser detection schemes for CDMA systems in the last two decades to improve the system capacity. The main reason that these schemes are not used in practical systems is because of their stringent processing requirements. Multiuser detection schemes are specified as optional [3] even in the proposals for the third generation wireless systems.

The complexity of the optimal multiuser detector (MUD) [4] increases exponentially with the number of users. Hence, several low-complexity sub-optimal schemes have been proposed and evaluated in the literature [5], [6]. The MUD schemes researched so far can be broadly classified into linear schemes and interference cancellation (IC) methods. Linear schemes require more computation but are robust against near-far resistance. Therefore, we focus on these schemes in this paper. In particular, we present results for schemes that are based on zero forcing (ZF) linear multiuser detection. ZF MUD schemes have lower complexity, as they do not require channel gain estimates.

The first linear ZF multiuser detection scheme was proposed in [7] for a synchronous CDMA system. Unfortunately, commercial cellular systems do not have synchronous uplinks since users transmit from random locations at random instants of time. Even if the users time their transmission such that the line-of-sight arrivals are aligned at the base station, the multipath rays render the system asynchronous. Thus it is necessary to consider asynchronous multiuser detection schemes in a practical situation. For asynchronous systems, the MUD proposed in [8] is difficult to implement as its complexity depends on the product of the number of users and the number of symbols in the whole duration of transmission. Several linear MUD schemes for asynchronous systems have since been proposed and analyzed. A survey of such schemes can be found in [9], [10] and the references therein. All these schemes are based on full symbol correlation detection at the base station.

More recently, techniques that consider sub-symbols for partial correlations have been proposed [1]. This paper focuses on fine tuning the sub-symbol technique and maximizing the number of simultaneous users for the combining algorithms presented in [11]. We assume that moderate synchronism can be achieved by issuing downlink commands to the users to adjust their transmission instants.

Section 2 briefly describes the system model considered in the sub-symbol scheme. In Section 3, we discuss the algorithms for sub-symbol combining and promote the idea that for a large number of users, one long sub-symbol is better than using several small sub-symbols. Simulation and analytic results and guidelines on choosing the optimum thresholds for the algorithms used in the sub-symbol scheme are presented in Section 4.

II. System model and sub-symbol scheme

The baseband model of a CDMA uplink with single-user matched-filter (SUMF) receiver is depicted in Fig. 1 for N_u users. The user data signals before spreading are denoted as $d_1(t), d_2(t), \ldots, d_{N_u}(t)$. These are multiplied by the spreading codes $K_1(t), K_2(t), \ldots, K_{N_u}(t)$ before transmission. The signal received at the base station is the sum of individual signals from each user modified by the respective channels, and the additive white gaussian noise. Succinctly, the received signal is given by:

$$r(t)=\sum_{j=1}^{N_u} K_j(t-\tau_j)A_j(t-\tau_j)d_j(t-\tau_j)+n(t)$$

where $\tau_1,\tau_2,\ldots,\tau_{N_u}$ are the delays incurred by individual users in the flat[1] wireless channels with single tap gains $A_1(t),A_2(t),\ldots,A_{N_u}(t)$. It is assumed that the variance of AWGN $n(t)$ is σ_n^2. The single-user matched-filter (SUMF) receiver performs a full-symbol correlation for each user independently with the output for the i^{th} user given by:

$$y_i(t)=\int_{symbol\ period} r(t)K_i(t-\tau_j)$$

To appreciate the use of ZF MUD in a sub-symbol scheme, which is inherently synchronous, the above expression for a synchronous system becomes:

$$y_i(t)=\sum_{j=1}^{N_u} R_{ij}(t)A_j(t)d_j(t)+n_i(t)$$

where $R_{ij}(t)$ is the crosscorrelation between the sequences $K_i(t)$ and $K_j(t)$, and the additive noise $n_i(t)$ has a variance $R_{ii}(t)\sigma_n^2$. It is important to note that additive noise in $y_i(t)$ is uncorrelated with the additive noise in $y_j(t)$ for $i\neq j$. The vector signal formed from the demodulated signal for individual user can be represented as:

$$\vec{y}(t)=\mathbf{R}(t)\mathbf{A}(t)\vec{d}(t)+\vec{n}(t)$$

It is apparent that slicing performed on $\mathbf{R}^{-1}(t)\vec{y}(t)$ would result in almost error free detection, save for the additive noise. This is precisely what the ZF Linear MUD does. It is easy to see that this operation results in elimination of MAI at the cost of increased power of additive noise. Simulation results for the ZF linear MUD have been reported in [12] and a detailed analysis can be found in [7] and [9].

If the system becomes asynchronous due to either multipath or random transmission instants from randomly located users, the size of the decorrelation matrix increases making it difficult to implement the MUD [8]. Techniques like isolation bit insertion [13] and sliding window decorrelator [9] have been proposed to limit the size of decorrelation matrix.

In the scheme presented in [1], the symbols for each user are broken into sub-symbols at the boundaries defined by the start and end of symbols from other users. This makes an asynchronous system look like a synchronous one with variable length symbols. The decorrelation operation is then carried out in each sub-symbol interval. This yields multiple estimates for the same symbol, one estimate for each of the sub-symbols in which decorrelation is performed. These estimates are combined according to the algorithm described in the next section.

[1] For frequency-selective channel, the received signal is modified appropriately.

III. NUMBER OF SUB-SYMBOLS

The decision to perform decorrelation in a particular sub-symbol is based on two thresholds defined in [1]. These two thresholds are central to the idea of sub-symbol diversity combining and are defined below:

(a) Chip Threshold (γ_c) which indicates that the correlation on a particular sub-symbol should only be performed if the size (in number of chips) of the sub-symbol exceeds this threshold. This can be used to eliminate cases where the number of users is greater than or equal to the number of chips in a sub-symbol.

(b) Condition Number Threshold (γ_n) which indicates that the correlation for a particular sub-symbol should not be performed if the correlation matrix during that sub-symbol is badly scaled. This is used to avoid the cases where the correlation matrix accidentally becomes ill-conditioned even though the number of chips over which correlation is performed is large.

The algorithms used for sub-symbol combining are either *simple* combining algorithms or *condition-number based* combining algorithms. In each algorithm, the sub-symbols are combined based on a diversity principle for each symbol from each user.

In [1] it is assumed that user transmission is uncontrolled in the sense that the starting instants of symbols received at the base station are uniformly distributed over the whole symbol period. For a small number of users, the sub-symbol intervals are long enough to allow easy decorrelation. But as the number of users increase, the expected size of the sub-symbols becomes smaller, and it becomes harder to perform decorrelations within sub-symbols. When the number of chips in a sub-symbol falls below the number of users, it is impossible to decorrelate all the users successfully in that sub-symbol since the decorrelation matrix always has one or more zero eigenvalues. In particular, if N_c is the number of chips in a given sub-symbol, the maximum number of linearly independent users (see [7] for details) in that sub-symbol is also N_c. For the correlation matrix, a regular inverse does not exist and a generalized inverse (or pseudo inverse) has to be used, while setting γ_n to infinity. Towards this end, it is important to distinguish between two types of sub-symbols for a given user, a *clean* sub-symbol indicating that this particular user has been successfully decorrelated in this particular sub-symbol and a *corrupted* sub-symbol, which corresponds to one of the zero eigenvalues of the correlation matrix.

When the correlation matrix is not of full rank, the maximum number of *clean* sub-symbols will be equal to the rank of the correlation matrix. Although one 'long' sub-symbol in which decorrelation can be performed is good enough to bring the bit error rate (BER) down by a significant amount (see select-best algorithm in [1]), combining many sub-symbols brings down the SNR gain. Combining the *corrupted* sub-symbols with the *clean* sub-symbols, however, deteriorates the BER, and it would be better not to combine those sub-symbols at all. Simulations for a sub-symbol scheme in which users transmission instants are uncontrolled indicate that combining all sub-symbols greater than a relatively low γ_c (less than 5% of the spreading gain) would result in the allowable number of users only up to 8-9% of the spreading gain. This is also indicated in Fig. 2 for a spreading gain of 255 with Kasami sequences [14]. As more and more users are packed, sub-symbols become smaller and smaller and for a given user many of the sub-symbols are *corrupted* which significantly affect the BER performance.

In this paper, we consider a moderately controlled environment in which the users are instructed[1] to adjust their transmission instants such that their symbol arrival times at the base station fall within some fraction of the symbol period. This kind of moderate synchronization is easier to implement as compared to a fully synchronized system. The thresholds for sub-symbol decorrelation then play an important role in determining the maximum number of users that can be simultaneously accommodated in the system. The chip threshold (γ_c) is set high enough such that only one sub-symbol is used for demodulation. This has an effect on processing gain, but if the users can be decorrelated successfully, it still yields a significant BER improvement over the single-user matched-filter receiver for medium to high signal to noise ratios. The sub-symbol combining system thus becomes the one that only uses selection sub-symbol diversity since we consider only one *dominant* sub-symbol per symbol. Thus all the sub-symbol diversity-combining algorithms converge to the "select best sub-symbol" combining algorithm. The only difference is that, in this case, it is known which sub-symbol would be selected for decorrelation.

The SNR penalty for such an algorithm with respect to an ideal case can be easily computed. Assume that *S* denotes the total signal, *X* denotes the total noise and *Z* represents the noise per chip in the decorrelation interval. Also let *N* be the spreading factor (chips per symbol). The sampling is assumed to be one sample per chip and it is also assumed that fading imposes a fixed penalty in each receiver. These assumptions can be justified for comparison of different receivers since they are used in similar environments. The subscript "*f*" is used to refer to the case of full-symbol synchronous and the subscript "*s*" is used to denote the case of sub-symbol synchronous receiver. Constant fading penalty assumption allows comparing non-faded systems. For full-symbol synchronous system, the decorrelation is performed over one full symbol and we can write:

$$S_f = \sum_{i=1}^{N} 1 = N$$

$$X_f = X_1 + X_2 + \cdots + X_N$$

where the X_i's are i.i.d. Normal$(0, \sigma_n^2)$ random variables, and indicate the chip noise. Hence,

$$\sigma_{X_f}^2 = N\sigma_{X_1}^2$$

$$Z_f = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$\sigma_{Z_f}^2 = \sigma_n^2 / N$$

For the sub-symbol case, let *M* be the number of chips in the best symbol, as selected by the select-best algorithm. If only one sub-symbol is used, then *M* is equal to the expected number of chips in the longest sub-symbol within a symbol. *M* can be analytically computed or it can be found from simulations. As an example, for 12 asynchronous users with uniformly distributed random transmission instants, *M* turns out to be approximately 55 for the case *N*=255. The general procedure for computing *M* when user transmission instants are moderately synchronized will be outlined later. Similar to the above computations, for the sub-symbol of length *M*, we can write:

$$S_s = \sum_{i=1}^{M} 1 = M; \qquad X_s = X_1 + X_2 + \cdots + X_M$$

$$\sigma_{X_s}^2 = M\sigma_{X_1}^2; \quad Z_s = \frac{1}{M}\sum_{i=1}^{M} X_i; \quad \sigma_{Z_s}^2 = \sigma_n^2 / M$$

Thus the SNR penalty (deterioration in SNR with respect to ideal ZF MUD) for the select-best algorithm in approximately given by *N/M*. This penalty can be reduced if fewer users are present and more sub-symbols can be combined. If the system is moderately synchronized and the users are allowed to have their transmission instants lie within the first *P* chips of a reference symbol, then the expected value of *M* is given by:

$$E[M] = (N-P) + \sum_{i=0}^{P-1}\left\{i\left(\alpha^i - \alpha^{i+1}\right)\right\} + P\alpha^P$$

$$= (N-P) + \frac{\alpha - \alpha^{P+1}}{1-\alpha}$$

where α is defined as:

$$\alpha \equiv \left(\frac{P-1}{P}\right)^{N_u}$$

For practical reasons, *P* is a large fraction of *N* (typically 50% to 80%) in which case the expected value of *M* for a large number of users is approximately given by *N-P*. The exact expected value is listed in the following table for *N*=255.

	P = 64	P = 128	P = 192
N_u = 10 users	196.8627	139.2559	81.6535
N_u = 30 users	192.6558	130.7696	68.8964
N_u = 50 users	191.8349	129.0826	66.3517
N_u = 70 users	191.4972	128.3669	65.2661
N_u = 90 users	191.3199	127.9750	64.6668

[1] Sending commands on the downlink channel can accomplish this.

IV. Optimizing the Thresholds

For the decorrelation process in each sub-symbol, either the regular inverse (which we will denote by $\Im$) can be used or the generalized inverse (denoted by $\wp$) can be used. We will consider the effect of thresholds in each case. The condition number threshold γ_n is important only if the $\Im$ operation is performed and does not have much significance for the $\wp$ operation. As long as the number of chips in a sub-symbol is less than the number of users in that sub-symbol, $\Im$ operation can not be performed. Selecting γ_c as less than the number of users would result the performance of $\Im$ operation to become independent of γ_c. As the number of users increases, it may not be possible to perform the $\Im$ operation at all and $\wp$ is the only recourse. In such a case, a small value of γ_c results in more and more sub-symbols being considered for decorrelation, which increases the probability of contamination of clean sub-symbols by the corrupted sub-symbols. Thus increasing the number of users brings two benefits: First, more users are accommodated and secondly there is less chance for a corrupted sub-symbol being considered for combining with closely packed users. Therefore, if users transmission instants can be bound within a small region, a value of γ_c exceeding the length of that region is desirable. Otherwise, with fewer users, it is better to use the $\Im$ operation.

If the $\wp$ operation is to be used, then it should be made sure that in each sub-symbol, there are enough chips to decorrelate the many users. If most of the users in a sub-symbol remain dependent, they will deteriorate the performance achieved by the rest of the sub-symbols, and it is advantageous to isolate that sub-symbol. Obviously, the ideal situation is to be able to *pick* the sub-symbols for only those users that have been decorrelated successfully. If it is not possible to selectively pick users from within a sub-symbol, the chip threshold γ_c should be selected such that there is a good chance for most of the users to be decorrelated. Thus as the number of users increases, a larger value of γ_c is needed, upto a certain limit, after which it should be set to a constant value such that we have at least one sub-symbol to carry out the decorrelation process.

An alternate measure for performance adjustment is the rank threshold γ_r, which indicates that the decorrelation operation is performed in a sub-symbol if the corresponding correlation matrix has a rank that exceeds the rank threshold. For small number of users, γ_r should be made equal to the number of active users or even slightly higher in order to eliminate the risk involved in considering a corrupted sub-symbol for later combining. Under this condition, γ_n can be used in conjunction with γ_r to great advantage. For a very large number of users, we have only one sub-symbol available that *has* to be decorrelated and a value of γ_r slightly smaller than the number of users would be the right choice. Under the above rules, the maximum number of users in a moderately synchronized system with a spreading gain of 255 is given in the following table:

P = 128 chips, γ_c = 25% of #users		
Pinv ($\wp$)	Inv ($\Im$) (γ_n = 200)	Inv ($\Im$) (γ_n = 500)
116-118	97	110

P = 192 chips, γ_c = 100% of #users		
Pinv ($\wp$)	Inv ($\Im$) (γ_n = 200)	Inv ($\Im$) (γ_n = 500)
62-63	49-52	51-54

The maximum number of supported users shown in the above table are determined by the criteria that the sub-symbol detector results in a bit error rate (BER) that is at least an order of magnitude better than the single-user matched-filter receiver at a signal to noise ratio of 30dB.

V. Conclusions

We presented results on the maximum number of simultaneous users that can be supported in a DS-CDMA system under the sub-symbol scheme when moderate synchronization can be enforced among the users. Under such conditions, it is shown by simulations that only one sub-symbol is enough for accommodating a fairly large number of users. It turns out that by using the appropriate values for the chip and condition number thresholds, the "select best" sub-symbol scheme can support a maximum number of users that is approximately given by the maximum number of consecutive chips in any sub-symbol. For example, if the longest sub-symbol is 63 chips long, irrespective of the spreading factor, the maximum number of users supported is approximately 63. In this case, for each user, the sub-symbol based MUD yields a BER that is at least an order of magnitude better than a single-user matched-filter detector.

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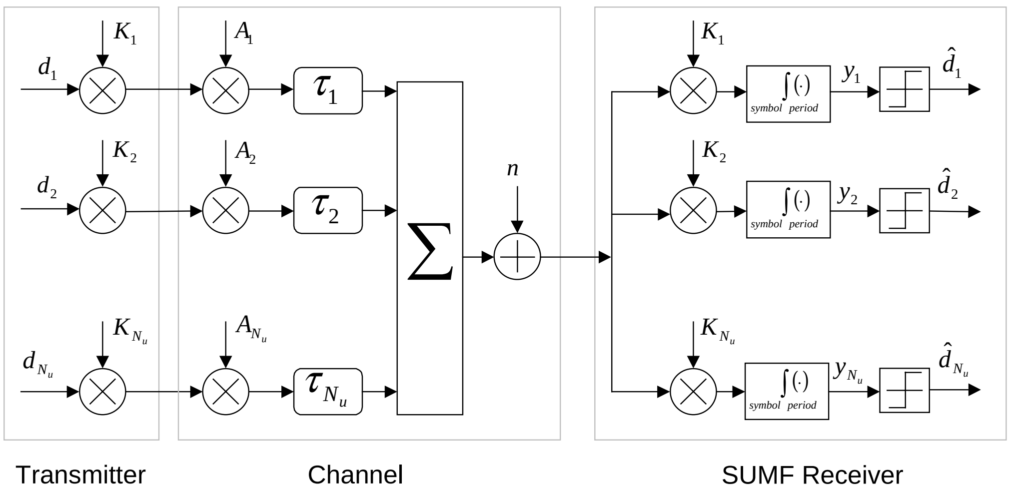

Fig. 1: Baseband DS-CDMA System Model with SUMF Receiver.

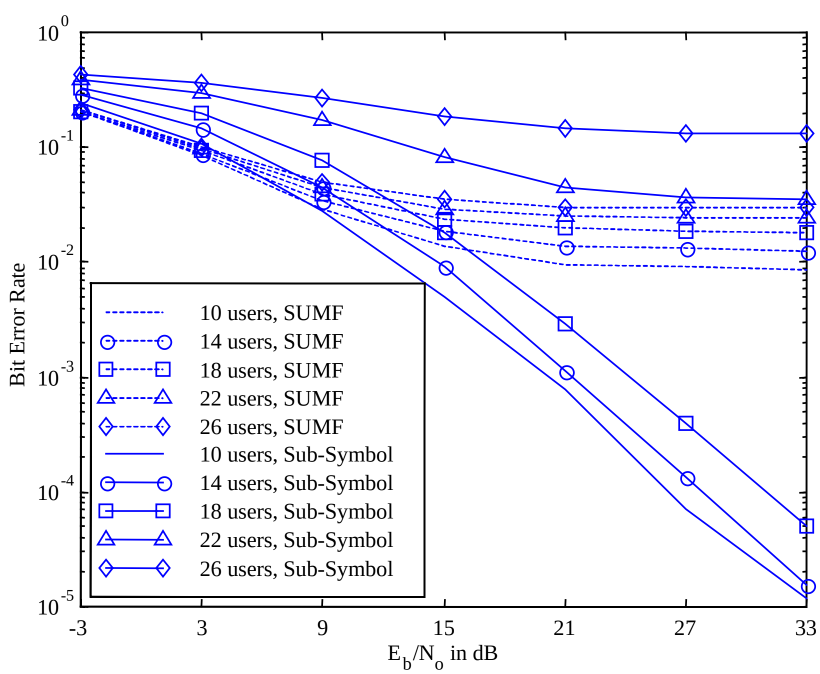

Fig. 2: Maximum Number of Users in Sub-Symbol Schemes with $\gamma_c=1.25N_u$ and $\gamma_n=\infty$ for completely asynchronous system.