A NOVEL SCHEME FOR LINEAR MULTIUSER DETECTION IN ASYNCHRONOUS DS-CDMA SYSTEMS WITH FREQUENCY SELECTIVE FADING

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Abstract

A *novel* scheme for linear Multiuser Detection (MUD) based on *Sub-Symbol* diversity is presented that is applicable to Asynchronous CDMA systems with frequency selective fading. Traditional solutions for asynchronous linear MUD result in a prohibitively large decorrelation matrix. The new scheme transforms the asynchronous problem into a synchronous one by creating sub-symbols that are, by design, time-aligned with all other users and their respective multipath signals. The resulting decorrelation matrix is dramatically reduced in size, thus offering a viable solution for implementation. We also present an optimal scheme for combining the sub-symbols that is based on the condition number of the decorrelation matrices.

1. Introduction

Multiple Access Interference (MAI) is known to limit the capacity of a DS-CDMA system. The conventional detection scheme, using a single-user matched-filter (SUMF) in a DS-CDMA base-station receiver, ignores the MAI completely. Therefore, the bit error rate (BER) of a particular user depends on the powers of signals received from other users. This results in a poor BER performance in a mobile communications environment.

The demodulated information from all users can be jointly processed to improve the BER performance. One of the earliest multiuser detectors for a CDMA system was derived in [1]. The optimum multiuser detector (MUD) was derived in [2], but it is too complex to implement. A number of sub-optimum low-complexity schemes have since been proposed and evaluated extensively (see [3], [4] and the references therein).

The zero forcing (ZF) linear MUD attempts to completely eliminate MAI without any consideration for AWGN, and is known to be optimally near-far resistant. In a ZF MUD, demodulated signals from all the users are passed through a linear filter that can be represented as a matrix [5], called the decorrelating matrix. For synchronous CDMA systems with flat fading, the size of the decorrelating matrix is $N_u \times N_u$ where N_u is the number of active users. Commercial CDMA systems are not synchronous in general. For asynchronous systems, the ZF MUD becomes significantly complex [6] even with flat fading, as an $N_s N_u \times N_s N_u$ matrix is needed where N_s is the number of symbols in the whole duration of transmission. The technique of Isolation Bit Insertion (IBI) [7] can be used to limit the size of the decorrelating matrix at the cost of reducing the effective data rate and requiring moderate synchronism between the users.

For frequency-selective synchronous CDMA channels, a ZF MUD as proposed in [8] requires an $N_r N_u \times N_r N_u$ decorrelating matrix where N_r is the number of resolvable paths in the multipath channel. This detector is modified in [9] to reduce the decorrelating matrix size to $N_u \times N_u$. Both of these schemes for a frequency-selective fading channel work on a single-shot basis, and hence the multipath from previous symbols contaminating the current symbol is neglected.

A detection scheme is, therefore, needed that should be able to accommodate asynchronous users and their multipaths while being optimally near-far resistant. Further, such a scheme should be implementable with reasonable complexity and be independent of the number of symbols in a transmission without degrading the symbol rate.

In this paper, we present a novel scheme that works by exploiting a new form of diversity called *sub-symbol diversity* to achieve multiuser detection in an asynchronous CDMA system with a plurality of resolvable paths from each user. It is shown that the new scheme represents a generalized implementation of the RAKE receiver [10] currently used in an asynchronous systems with multiple resolvable paths.

2. Transmitter and Channel

The signal received at the base station is a summation of all the user signals, their respective multipaths, and additive white gaussian noise (AWGN). Consider N_u users where data signals from each user are designated as $d_1(t), d_2(t), \ldots, d_{N_u}(t)$ and the spreading sequences as $K_1(t), K_2(t), \ldots, K_{N_u}(t)$. The channel introduces delays $\tau_1, \tau_2, \ldots, \tau_{N_u}$ to signals from different users, and $A_1(t), A_2(t), \ldots, A_{N_u}(t)$ are the fading coefficients for the

[1] This research was done when author was visiting Bell Labs in 1998.

single resolvable path[1] of each user. At this point, it is convenient to define the symbol-period correlation matrix for the *time-aligned* spreading sequences of the active mobile users. Therefore,

$$R_{ij}(t) = \int_{symbol\ period} K_i(t) K_j(t)$$

is the element of symbol-period correlation matrix in the *i-th* row and *j-th* column.

The received signal $r(t)$ at the receiver front end can be written as:

$$r(t) = \sum_{j=1}^{N_u} K_j(t-\tau_j) A_j(t-\tau_j) d_j(t-\tau_j) + n(t)$$

where $n(t)$ is additive white gaussian noise (AWGN) with variance σ_n^2.

3. Existing receivers

Currently deployed DS-CDMA receivers use single-user matched-filter (SUMF) detection. An SUMF is essentially a correlation detector in which the correlation is performed over a symbol period. In a particular symbol, the output of the SUMF is given by:

$$y_i(t) = \int_{symbol\ period} r(t) K_i(t-\tau_j)$$

which, for a synchronous system, reduces to

$$y_i(t) = \sum_{j=1}^{N_u} R_{ij}(t) A_j(t) d_j(t) + n_i(t)$$

where $n_i(t)$ is the gaussian noise with variance $R_{ii}(t)\sigma_n^2$, $R_{ii}(t) A_i(t) d_i(t)$ is the desired signal, and the rest of the terms account for multiple access interference (MAI) which severely degrades the performance and limits the capacity of a DS-CDMA system [11].

In our model, signals from all the users at the base station can be represented in a vector form as follows:

$$\mathbf{y}(t) = \mathbf{R}(t)\mathbf{A}(t)\mathbf{d}(t) + \mathbf{n}(t)$$

where the time dependence is from symbol to symbol. In this equation $\mathbf{R}(t)$ is the symbol-long correlation matrix and $\mathbf{A}(t)$ is a diagonal matrix of fading coefficients. The slicing operation on $\mathbf{y}(t)$ includes MAI in the decision (the SUMF detector for each user) while the slicing operation performed on $\mathbf{R}^{-1}(t)\mathbf{y}(t)$ yields the correct decisions except for the additive noise. The decorrelating matrix size for a synchronous system with flat fading is $N_u \times N_u$. For asynchronous systems with flat fading, the ZF decorrelation can be performed (see [6]) but the decorrelating matrix becomes $N_s N_u \times N_s N_u$ in size.

4. The New Scheme

All the existing detection schemes are based on symbol-period correlation performed at the receiver for each of the user. We propose a partial-symbol correlation detector. It will be shown that performing partial-period correlations according to a specific algorithm allows the asynchronous system to be treated like a synchronous system with variable length symbols. The Zero Forcing linear decorrelation can then be performed on such a system.

4.1 Architecture

The scheme operates by slicing each symbol into at most N_u sub-symbols. This partitioning requires knowledge of the symbol boundaries[2] of all other users. It should be noted that multiple resolvable paths from each user are initially treated as separate users and then combined after sub-symbol combining has taken place. Fig. 1 assumes two asynchronous users whose symbol boundaries are known, and thus each symbol is broken into two sub-symbols. Every sub-symbol for a particular user is delimited by boundaries that mark the start of new symbols for any two consecutive users. The correlation operation is performed on a sub-symbol-by-sub-symbol basis, and the result stored for later combining with the other sub-symbols originating from the same symbol. In Fig. 1, each of the four sub-symbols from t=0 to t=t_2 can be mathematically represented as:

$$y_1^1 = \int_0^{t_1} r(t) K_1(t-\tau_1) dt, \quad y_1^2 = \int_{t_1}^{t_2} r(t) K_1(t-\tau_1) dt$$

$$y_2^1 = \int_0^{t_1} r(t) K_2(t-\tau_2) dt, \quad y_2^2 = \int_{t_1}^{t_2} r(t) K_2(t-\tau_2) dt$$

where y_1^1 and y_1^2 belong to the *same* symbol of first user and y_2^1 and y_2^2 belong to *different* symbols of the second user. It is important to note that y_1^1 and y_2^1 contaminate each other as do y_1^2 and y_2^2. Therefore, the decorrelation operation is accordingly performed within each sub-symbol as indicated in Fig. 1 where the decorrelating matrices R_1 and R_2 are given as follows:

$$R_1 = \begin{bmatrix} \int_0^{t_1} K_1 K_1 & \int_0^{t_1} K_1 K_2 \\ \int_0^{t_1} K_2 K_1 & \int_0^{t_1} K_2 K_2 \end{bmatrix}; \quad R_2 = \begin{bmatrix} \int_{t_1}^{t_2} K_1 K_1 & \int_{t_1}^{t_2} K_1 K_2 \\ \int_{t_1}^{t_2} K_2 K_1 & \int_{t_1}^{t_2} K_2 K_2 \end{bmatrix}$$

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[1] The product with fading coefficients is equivalent to a time varying convolution with a single tap filter response and corresponds to a single resolvable path. For multiple resolvable paths, the product operation is replaced by more general time-varying convolution with multiple taps, where each tap is individually faded and corresponds to one resolvable path.

[2] "Search and Acquisition" algorithms are used to obtain the symbol boundaries.

The inputs to each of these decorrelating filters are generated at the same time and no delay is required to perform the decorrelating operation within each sub-symbol. However, different decorrelating filters (R_1 and R_2 in the above case) are not synchronous with each other. Therefore, all the inputs to each combiner should be synchronized for proper combining by introducing appropriate delays.

It is clear from above description that each symbol undergoes multiple decorrelations, but the size of decorrelating matrices in each sub-symbol is just $N_u \times N_u$. Furthermore, the decorrelating matrices for each sub-symbol belonging to same symbol can be computed in parallel. Thus, with the new scheme, ZF multiuser detection can be performed with a much smaller decorrelating matrix and with no increase in bandwidth[1].

The architecture presented in Fig. 1 can be further simplified by noting that sub-symbols belonging to a particular symbol are non-overlapping. Thus, only one integrator is used for each user and the correlation product is dumped to the appropriate location at the end of each sub-symbol.

4.2 RAKE Implementation

RAKE receivers generally employ multiple detection fingers where each finger is a complete SUMF detector [12]. The new scheme can be easily used to realize a RAKE receiver by considering each resolvable multipath as a different user, with one combiner for each multipath. Each combiner simply sums the signals at its input and generates a signal corresponding to signal at the output of one RAKE finger. Furthermore, different combiners corresponding to a symbol and its multipath can jointly process the information disintegrated into sub-symbols rather than symbols.

4.3 Stability

It was noted in [5] that the docorrelation matrix is obtained by taking the Moore-Penrose generalized inverse of the correlation matrix. If the correlation matrix is badly scaled, the ZF MUD operation can't be performed and the performance approaches that of a single-user matched-filter. It was further noted in [13] that a high condition number of correlation matrix results in a poorer BER performance of the ZF MUD.

The scheme is thus based on two thresholds as follows:

Chip Threshold (γ_c) which indicates that the correlation on a particular sub-symbol should not be performed if the size (in number of chips) of the sub-symbol does not exceed this threshold. This can be used to eliminate all such cases in which the number of users is greater than or equal to the number of chips in a sub-symbol.

Condition Number Threshold (γ_n) which indicates that the correlation for a particular sub-symbol should not be performed if the correlation matrix during that sub-symbol is badly scaled. This is used to avoid the cases of correlation matrix accidentally becoming ill-conditioned even with large number of chips over which correlation is performed.

It is clear that the two types of asynchronisms that arise are due to random transmission instants/distances of users and due to frequency selective fading. The former is controllable while the latter has to be accounted for by the detection scheme. Since the new scheme counts on the chip threshold, the more users are synchronous, the more chips are likely to be in a sub-symbol and the better the scheme works. The scheme only needs one "long" sub-symbol with easily invertible correlation matrix to completely eliminate MAI. For simulations, we considered worst case asynchronous users in which case the symbol starting instants of users are independently uniformly distributed (no effort is made to synchronize the first resolvable path from different users). If the system is synchronous with flat fading, there is just one sub-symbol per symbol, and the scheme achieves the best performance ZF MUD can achieve for such a system.

4.4 Combining algorithms

In this paper, we will consider two sets of sub-symbol diversity combining algorithms, namely the *simple* combining algorithms and the *condition number based* algorithms. The *simple* combining algorithms are similar to the ones used for combining the signals at the RAKE fingers [12]. The difference being that RAKE combining algorithms combine signals from different resolvable paths while in the new scheme, the combining is done for sub-symbols. Three *simple* algorithms for sub-symbol diversity combining are as follows:

- *Select best*: In this algorithm the sub-symbol with maximum amplitude is selected.
- *Weighted*: In this case, each sub-symbol is weighted by the strength of the signal within that sub-symbol.
- *Accumulate*: This algorithm simply sums the signal in all the sub-symbols belonging to a particular symbol and decision is made on the accumulated result.

In addition to the *simple* combining algorithms, the scheme lends itself to define a new family of algorithms since the signals are obtained from different sub-symbols which, unlike RAKE fingers, are obtained by performing correlations over different number of chips. For example, in *condition number based* combining algorithms, the simple combining algorithms stated above are modified such that each sub-symbol is weighted by the inverse condition number of the decorrelation matrix during that

[1] Isolation Bit Insertion (IBI) increases the decorrelating matrix size and reduces bandwidth. See [7] for details.

sub-symbol. This condition-number based combining is inferred from the result presented in [13] that the BER performance of ZF MUD deteriorates with increasing condition number of the correlation matrix. Such algorithms exhibit better BER performance than simple combining algorithms as indicated in the next section.

5. Performance Results

Zero Forcing (ZF) Linear MUD is known to completely eliminate MAI and is optimally near-far resistant (see [5], [6], [13]). The new scheme presented in this paper employs the Zero Forcing principle in each sub-symbol to eliminate MAI and thus performs significantly better than a single-user detector. The performance margin clearly depends upon the specific combining algorithm used. For *simple* sub-symbol diversity combining algorithms (similar to the ones used in combining multipath diversity [12]), the results are as depicted in Fig. 2. Of all the three *simple* combining algorithms, the accumulate algorithm exhibits the best BER performance, because it tends to average the noise over the whole symbol. Since ZF MUD is known to perform poorly at low SNRs, the three *simple* combining algorithms also show poor performance at low SNRs because they are based on zero forcing principle. Furthermore, Fig. 2 indicates that the *simple* combining algorithms perform 4dB to 8dB worse than an ideal ZF MUD (used in completely synchronous system) for uncoded BERs of 10^{-2} or better.

Since the ZF MUD performance is dependent on the condition number of the decorrelation matrix [13], it is possible to do better than the *simple* algorithms. This is accomplished by using an algorithm that weighs the sub-symbols in inverse proportion of the condition numbers of correlation matrices within those sub-symbols. Fig. 3 depicts the BER performance of such an algorithm compared with that of a *simple* accumulate combining algorithm, and it can be seen that the *condition number based* algorithm performs 3dB better than the *simple* accumulate combining algorithms for 10^{-3} or better BERs.

6. Conclusions

We have presented a novel scheme for linear multiuser detection in a DS-CDMA system. The new scheme facilitates decorrelating the undesired users from the received signal at the base-station receiver. The new scheme keeps the linear decorrelating filter computationally tractable while accommodating asynchronicity among users created by either random transmission instants or multiple resolvable paths from each user. Unlike previously known multiuser detection schemes for asynchronous systems, this scheme does not depend on the number of symbols in the duration of transmission and it does not reduce the symbol transmission rate.

The scheme reveals a new kind of diversity named as sub-symbol diversity. Some diversity combining algorithms for sub-symbol diversity have also been presented in this paper. Simulations confirm that the new scheme is near far resistant and performs significantly better than a single-user matched-filter detector.

The new scheme also represents an alternate construction of the RAKE receiver in which case users are not decorrelated within sub-symbols and sub-symbol diversity combiner is used as a multipath diversity combiner.

7. References

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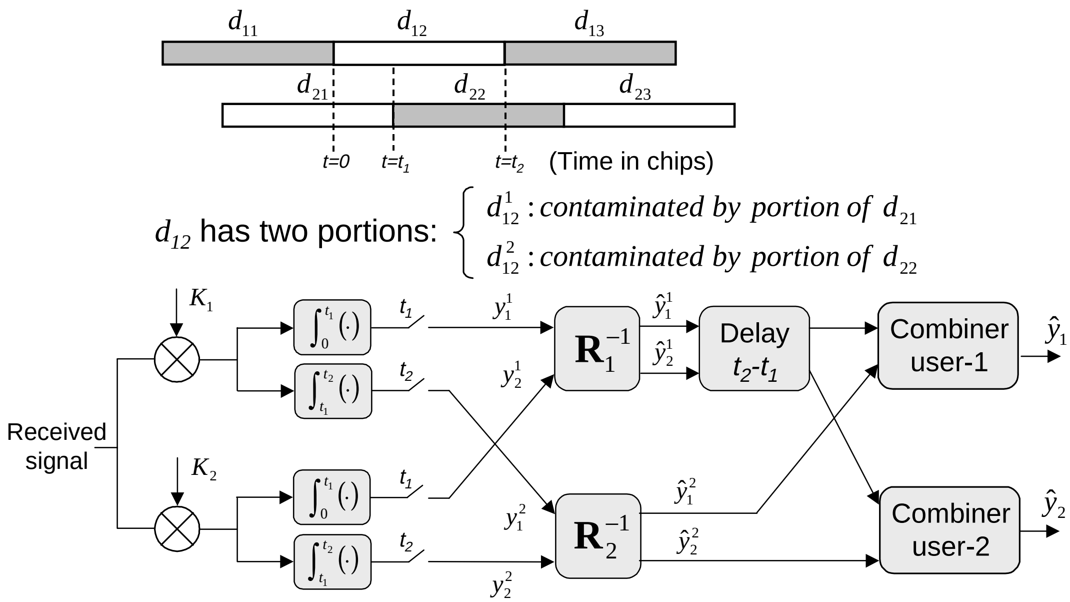

Fig. 1: New Detection Scheme for 2-User Case

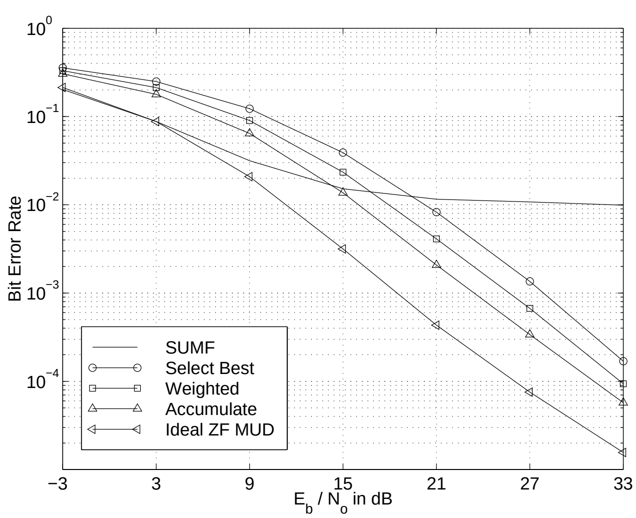

Fig. 2: Performance of New Scheme with *simple* diversity combining algorithms (N_u=12 users, condition number threshold γ_n=100, chip threshold γ_c=3 chips)

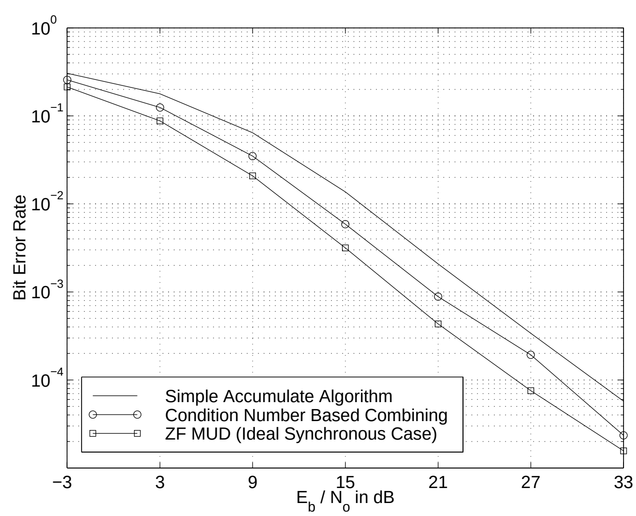

Fig. 3: Performance of *Condition number based* combining algorithm (N_u=12 users, γ_n=100, γ_c=3 chips)