Performance Analysis of a Convolutionally-Encoded Synchronous CDMA System with Adaptive Beamforming and Linear Multiuser Detection

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Abstract

Beamforming (BF) with multiple receiving antennas significantly improves the bit-error-rate (BER) performance for a DS-CDMA system in a flat Rayleigh fading environment. However, BF fails to suppress multiple access interference (MAI) completely, which results in an irreducible BER floor. Furthermore, soft-decision Viterbi decoding (VD) with BF further lowers the BER floor.

The BER-performance difference between the minimum mean square error (MMSE) linear multiuser detector (MUD) and Zero Forcing (ZF) MUD increases with the eigenvalue spread of the code correlation matrix. BF with MUD performs better than MUD alone, and BF+VD is much more effective at combating MAI than MUD+VD for low to moderate SNRs. In this regime, performance of BF+VD approaches that of BF+MUD+VD, which clearly offers the best performance for all SNRs.

1. Introduction

For optimal reception of all users in a DS-CDMA uplink, the spreading codes should be drawn from an orthogonal set. However, multipath in a mobile radio channel destroys orthogonality. As a result, the objective is to achieve a minimal cross-correlation among the different spreading codes. Currently deployed CDMA systems use single-user matched-filter (SUMF) detection of individual users, in which case signals from other users are regarded as noise. This approach severely limits the capacity of a DS-CDMA system [1]. A number of multiuser detection schemes have been proposed and analyzed in the last decade, where the focus has been on the architecture of detectors [2], [3], [4] and their near-far resistance properties [3], [5].

To assess the performance of multiuser detectors (MUDs) in realistic scenarios, it is essential to include channel coding. Furthermore, BER performance of MUD with multiple antennas and channel coding has not been shown previously. We provide a comprehensive comparison of BER performances of linear MUDs with multiple antennas and channel coding. It is also demonstrated that the difference in BER performance between ZF and MMSE MUDs is strongly dependent on the correlation matrix of the spreading codes.

2. Transmitter and System Model

For each of the N_u active users in the system, the data bits are convolutionally encoded and then spread by multiplying with a 255-chip long mobile station specific Kasami sequence [6]. These sequences are derived from a small set of 16 sequences and are designated as $\mathbf{K}_1, \mathbf{K}_2, \cdots, \mathbf{K}_{16}$. The correlation between two Kasami sequences $\mathbf{K}_i$ and $\mathbf{K}_j$ is denoted as R_{ij}, and since the system is assumed to be synchronous, this correlation is always computed over one symbol.

Signals from individual mobile stations are assumed to undergo flat Rayleigh fading with fading amplitudes $A_1, A_2, \cdots, A_{N_u}$ that are assumed to be constant over a symbol. The signal $\mathbf{r}$ received at the receiver front end is thus given by:

$$\mathbf{r} = \mathbf{K}_1 A_1 d_1 + \mathbf{K}_2 A_2 d_2 + \cdots + \mathbf{K}_{N_u} A_{N_u} d_{N_u} + \breve{\mathbf{n}}$$

where $\sigma_{\breve{\mathbf{n}}}^2$ is the additive noise variance in each chip.

The most basic CDMA receiver employs a single-user matched-filter (SUMF) or a correlator, in which each user is demodulated individually without any knowledge of signals from other users. The output of the single-user matched-filter for *i-th* user is given by:

$$y_i = \sum_{symbol\ \ period} \mathbf{r}\mathbf{K}_i^{\mathbf{T}}$$

For a particular user, received signal y_i can now be considered as a sum of three components:

$$y_i = \underbrace{R_{ii} A_i d_i}_{\text{desired signal}} + \underbrace{\sum_{\substack{j=1 \\ j \neq i}}^{N_u} R_{ij} A_j d_j}_{\text{MAI}} + \underbrace{n_i}_{\text{gaussian noise}}$$

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[1] This work was done when author was visiting Bell Labs in 1998.

The first component is the desired signal and is always a correct decision statistic for d_i. The second term in y_i is multiple access interference (MAI) and the third term is due to colored noise. MAI does not decrease with increasing signal-to-noise ratio (SNR). Hence an irreducible BER (i.e. BER floor) is obtained in the presence of non-zero MAI. The despreaded signal, y_i, is finally processed by a *soft-decision* Viterbi decoder (VD) to recover the original information bits.

3. Receiver with Multiple Antennas

Receivers with multiple antennas provide a means of performing *spatial filtering*, which can potentially offer increased signal to interference-plus-noise ratio (SINR) by increasing the relative antenna gain in the direction of the desired user. Thus, the notion of spatial filtering is synonymous with beamforming, in which case the tap weights assigned to each antenna dictate the antenna gain pattern. The beamforming mechanism is simply characterized by means of an *array response vector* and a beamformer tap *weight vector* (i.e. tap weights in the spatial filter).

The *array response vector* is an indication of how an array responds to a signal. The *weight vector* is dynamically determined based on different criteria. In our simulations, a narrow angle of arrival is assumed that can be estimated fairly accurately, such that the spatial fading over the antenna elements can be neglected [7], [8]. Under such conditions, the beamforming algorithm that maximizes the signal to interference plus noise ratio (SINR) can be used effectively [7]. If $\mathbf{v}$ is the array response vector for a particular user, then the transpose of a weight vector that achieves the maximum signal to interference plus noise ratio (SINR), is given as [7]:

$$\mathbf{w} = \mathbf{R}_{uu}^{-1}\mathbf{v}$$

where $\mathbf{R}_{uu}$ is the correlation matrix of the undesired signal for that particular user. The undesired signal for a user consists of additive gaussian noise and the MAI for that user. A simpler structure for beamforming is to use the delay-and-sum algorithm, in which case the weight vector is the conjugate transpose of the array response vector. Thus delay-and-sum algorithm only compensates for the delays incurred in travelling from one antenna to the other.

Receivers with multiple antennas perform significantly better than a single antenna system but a BER floor is still observed, as MAI is not completely eliminated by using multiple antennas. Use of channel encoding with multiple antennas further improves the performance. However, a BER floor is again observed since Viterbi decoding is not designed to mitigate MAI.

For CDMA systems, beamforming can be performed either before or after the despreading. We simulate by first despreading followed by beamforming, since this allows to beamform at the symbol level.

4. Linear Multiuser Detection

Zero Forcing (ZF) linear MUD attempts to completely eliminate MAI in a synchronous CDMA system. The received signal after despreading can be written in matrix form as:

$$\mathbf{y} = \mathbf{RAd} + \mathbf{n}$$

where $\mathbf{R}$ is the Kasami correlation matrix, $\mathbf{A}$ is a diagonal matrix that contains Rayleigh fading coefficients for each user, and $\mathbf{d}$ is the vector of transmitted coded symbols. Since $\mathbf{R}$ is not a diagonal matrix, it reflects the MAI introduced due to non-zero cross-correlations. To suppress the MAI, ZF MUD performs the following operation [9]:

$$\mathbf{y}_{decor} = \mathbf{R}^{-1}\mathbf{y} = \mathbf{Ad} + \mathbf{R}^{-1}\mathbf{n}$$

Decisions made on signs of $\mathbf{y}_{decor}$ are always correct if additive noise term is neglected. However, for low signal-to-noise ratios (SNRs), the $\mathbf{R}^{-1}\mathbf{n}$ term becomes significant, resulting in poor performance. As indicated in Figs. 1 and 2, ZF MUD doesn't exhibit a BER floor since it completely eliminates MAI. However, ZF behavior leads to poor BER performance at very low signal-to-noise ratios. MMSE linear MUD accounts for the colored noise and uses $\left(\mathbf{R} + \sigma_n^2\mathbf{A}^{-2}\right)^{-1}$ as the decorrelating filter [10]**.** At high SNRs, performance of MMSE linear MUD converges to that of ZF linear MUD, whereas at low SNRs, the difference in performance depends on the following two parameters:

- Effective power in the second term of MMSE linear MUD filter (σ_n^2 dependent)
- Ratio of the max to min singular values of $\mathbf{R}$ (i.e. the condition number of $\mathbf{R}$)

For the set of Kasami sequences used, $\mathbf{R}$ exhibits a larger condition number if more sequences are selected from the set. For 12 users, ZF and MMSE MUDs perform almost alike. However, simulations with 15 users (see Fig. 2) show noticeable BER performance difference between the two MUDs.

5. ZF MUD with Beamforming

Beamforming and linear multiuser detection combat MAI, but suffer poorly in certain regions. ZF decorrelator MUD performs poorly for low signal-to-noise ratios, while a multiple antenna system with single user matched filter exhibits a BER floor at high signal-to-noise ratio (SNR). A combination of the two schemes offers substantial gains as pointed out in [11] based on architecture proposed in [12]. The complete receiver is shown in Fig. 5. At each element of the antenna array, individual users are first despreaded, then decorrelated via the $\mathbf{R}^{-1}$ filter and then beamformed individually. Such an approach completely eliminates MAI (and hence avoids a BER floor) at high SNRs while affording an adequate performance at lower SNRs, as shown in Fig. 3.

Performing the decorrelation operation at each antenna causes the undesired signal to appear as spatially white at the input of the beamformer, regardless of the angular size of the sector [11]. Thus for beamforming, delay-and-sum algorithm can be used instead of maximizing the SINR algorithm, without compromising BER performance.

6. Performance

Fig. 1 compares the SUMF and ZF MUD with and without beamforming and it clearly indicates that for BER specifications for voice and data transmission (i.e. 10^{-3} and 10^{-6} respectively), ZF MUD with BF outperforms all the other receivers.

Convolutional Encoding with Viterbi decoding (VD) significantly improves the BER performance on a single user communication channel [6], and a similar behavior is observed in CDMA system (see Fig. 3). The non-linear gain of VD with SNR results in some interesting observations. While Beamforming (BF) performs poorly on an unencoded channel as compared to BF+MUD, significant gains obtained from VD cause the BER performance of BF+VD approach that of MUD+BF+VD. An interesting finding is that BF+VD performs much better than MUD+VD for BERs greater than 10^{-5}, which is the BER floor for BF+VD. Furthermore, in this region, BF+VD performs almost as well as MUD+BF+VD. Thus, for moderate BER requirements (down to 10^{-5}), BF gain is much higher than MUD gain. Additionally, performance of MUD deteriorates significantly in the presence of *unknown* MAI (e.g. due to cochannel interference) [13] because it relies heavily on the knowledge of side information such as the number of active users and their spreading sequences. Delay-and-Sum Beamformer, on the other hand, does not require knowledge of other users' side information. Thus BF can mitigate known and unknown MAI almost equally well.

7. Near Far Resistance of Detectors

As discussed in Sec. 6, BF is more immune to unknown MAI as compared to MUD. MUD, on the other hand, is much more near-far resistant as compared to BF. Near-far problem arises when signals from mobile users reach the base station with unequal powers, and the user with lowest power suffers the highest BER. Detectors presented in this paper were analyzed for their near-far resistance and it was verified that ZF MUD is the most near-far resistant detector. To quantify the near-far resistance, simulations were performed by placing one weak user far from the base station, and then noting the BER of this user while other users progressively move closer to the base station. Fig. 4 clearly indicates that the SUMF and BF are not near-far resistant, and both result in an increase in the BER of weak user as other users move towards the base station. ZF and MMSE MUDs show extremely good near-far properties, because an undesired user with high er power appear as a single user to MUD, whereas it appears as an increased number of user to the BF. As indicated in Fig. 4, MMSE MUD has a lower BER than the ZF MUD for equal power users, but its BER converges to that of ZF MUD as the near users are moved nearer to the base station.

8. Conclusions

Zero Forcing (ZF) linear decorrelator with multiple antennas outperforms all the other architectures. Two main findings of this work are as follows:

For high SNRs, ZF and MMSE linear MUDs perform equally well, whereas for low SNRs, MMSE MUD outperforms ZF MUD by an amount that increases with the condition number of the correlation matrix of the spreading sequences.

With Viterbi decoding (VD), beamforming (BF) performs significantly better than multiuser detection (MUD) prior to reaching a BER floor of about 10^{-5}. While BF may appear to be a better choice than MUD, ZF MUD is more near-far resistant [3] than BF+VD.

Simulations of BER for a far (low SNR) user indicate that linear MUDs are extremely near-far resistant while BF is not. On the other hand, BF is robust against unknown MAI while MUDs are not. Thus a combination offers the best performance. For voice applications requiring BER of 10^{-3} or lower, the gain of ZF MUD+BF is 8dB more than a single-antenna ZF MUD (see Fig. 1)

With Viterbi decoding, an additional gain of 9dB is obtained. For data applications requiring BERs of 10^{-6} or lower, BF+MUD+VD remains the obvious choice.

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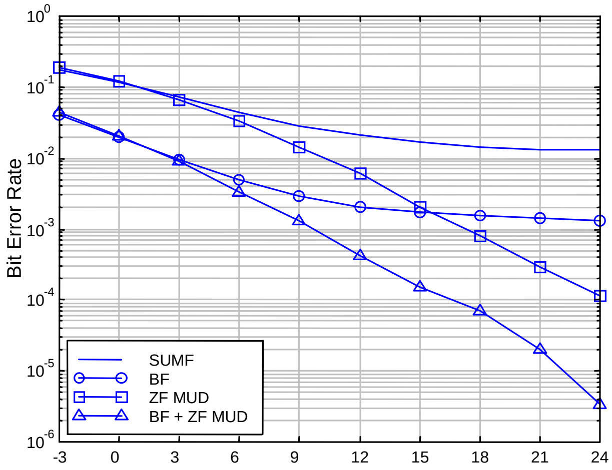

Fig. 1: Performance of ZF MUD with Beamforming for N_u=12 users and flat Rayleigh fading.

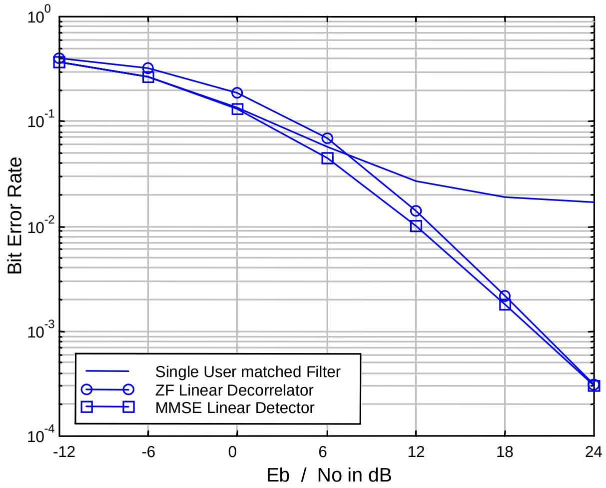

Fig. 2: Performance Comparison of ZF and MMSE Linear MUDs for N_u=15 Users and Single Antenna (For an unencoded channel).

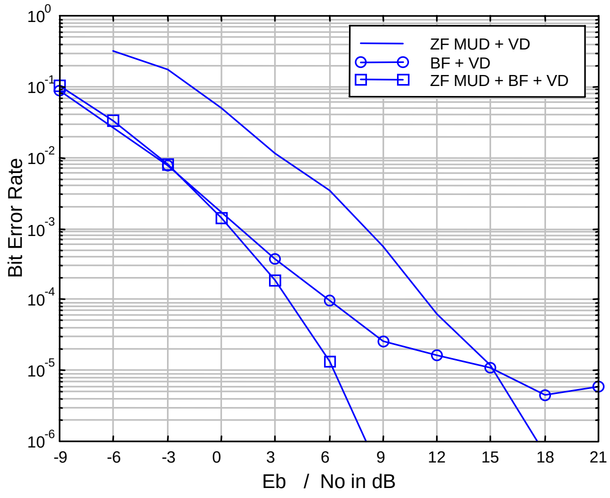

Fig. 3: Performance Comparison of ZF MUD and Beamformer for N_u=12 users

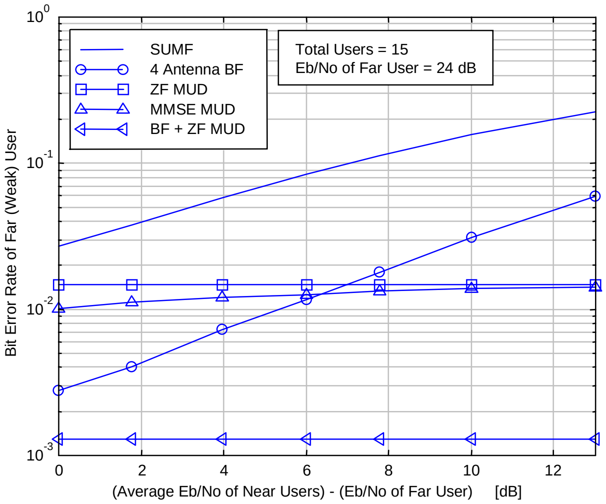

Fig. 4: Performance Comparison of ZF MUD and Beamformer for N_u=12 users

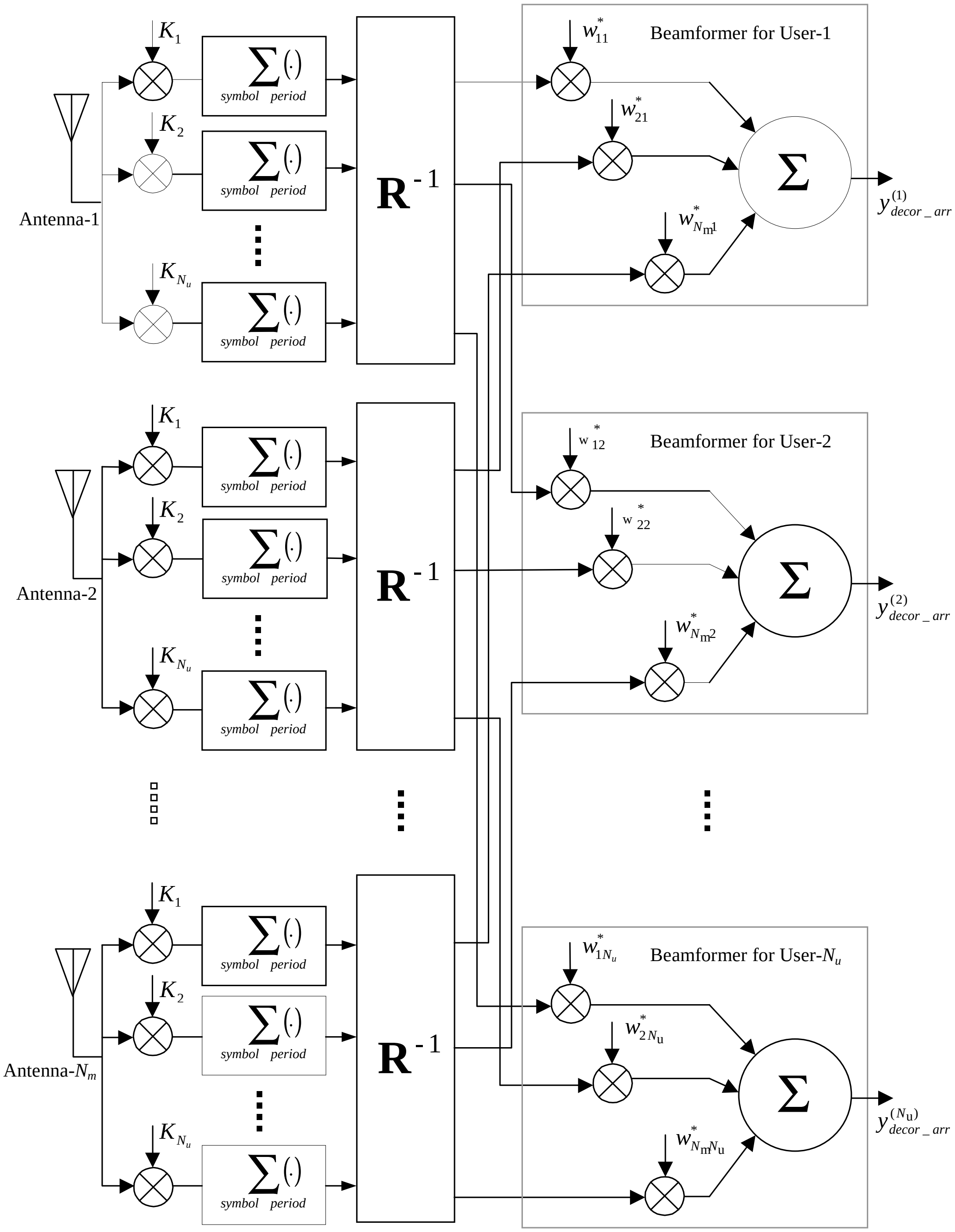

Fig. 5: Beamformer Receiver with ZF Linear Decorrelator