Reduction from RA to SC using fences

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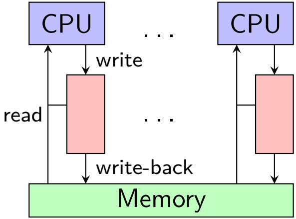
For TSO, it suffices to have a fence between every racy write & subsequent racy read.



For RA, we need more fences. Recall the IRIW example:

Independent reads of independent writes (IRIW)

$$x = y = 0$$

$$x := 1 \;\Big\|\; \begin{array}{l} a := x; \quad \textit{// 1} \\ b := y \quad \textit{// 0} \end{array} \;\Big\|\; \begin{array}{l} c := y; \quad \textit{// 1} \\ d := x \quad \textit{// 0} \end{array} \;\Big\|\; y := 1$$

What is the semantics of SC fences?

From C11, we had:

$$\mathtt{eco} \triangleq (\mathtt{rf} \cup \mathtt{mo} \cup \mathtt{rb})^{+} \qquad \text{(extended coherence order)}$$

$$\mathtt{psc}_{\mathtt{F}} \triangleq [\mathrm{F}^{\mathsf{sc}}]; (\mathtt{hb} \cup \mathtt{hb}; \mathtt{eco}; \mathtt{hb}); [\mathrm{F}^{\mathsf{sc}}] \qquad \text{(partial SC fence order)}$$

and required that $\mathtt{psc}_{\mathtt{F}}$ is acyclic.

That is,

Definition (RA consistency with fences)

An execution graph G is RA-*consistent* iff there exists some modification order $\mathtt{mo}$ for G such that:

- G is complete,
- $(\mathrm{po} \cup \mathtt{rf})^{+}|_{\mathtt{loc}} \cup \mathtt{mo} \cup \mathtt{rb}$ is acyclic, and
- $\mathtt{psc}_{\mathtt{F}}$ is acyclic.

Alternative definition of RA consistency

Theorem

An execution graph G is RA*-consistent iff there exists a total order* $\mathtt{sc}$ *on* $G.\mathtt{F}^{\mathit{sc}}$ *and a modification order* $\mathtt{mo}$ *for G such that:*

- *G is complete,*
- $(\mathrm{po} \cup \mathtt{rf} \cup \mathtt{sc})^+$ *is irreflexive, and*
- $(\mathrm{po} \cup \mathtt{rf} \cup \mathtt{sc})^*; \mathtt{eco}$ *is irreflexive.*

Theorem

Let G be an RA*-consistent execution graph. If*

- *For every G-racy events a, b, if $\langle a, b\rangle \in (G.\mathtt{po} \cup G.\mathtt{rf})^+$, then $\langle a, c\rangle, \langle c, b\rangle \in (G.\mathtt{po} \cup G.\mathtt{rf})^+$ for some fence event c.*

Then, G is SC-consistent.

Proof of the simple reduction theorem (1/2)

Recall:

- Recall SC-consistency : $\mathtt{po} \cup \mathtt{rf} \cup \mathtt{mo} \cup \mathtt{rb}$ is acyclic.
- Let $\mathtt{hb} \triangleq (\mathtt{po} \cup \mathtt{rf} \cup \mathtt{sc})^+$ and $K \triangleq \mathtt{eco} \setminus \mathtt{hb}$.
- It suffices to prove : $\mathtt{hb} \cup K$ is acyclic.

Consider minimal cycle in $(\mathtt{hb} \cup K)$.

- Cycles with ≤ 1 K-edges disallowed by RA consistency.
- Cycle with two K-edges:

$$\begin{tikzcd}
a \arrow[r, "K"] & b \arrow[d, "\mathtt{hb}"] \\
d \arrow[u, "\mathtt{hb}"] & c \arrow[l, "K"]
\end{tikzcd}$$

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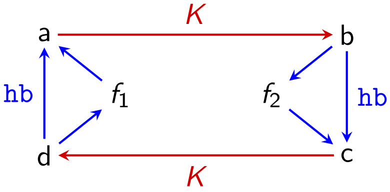

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$$
\begin{tikzcd}
a \arrow[rrrr, "K"] & & & & b \arrow[dl] \arrow[dd, "\mathtt{hb}"] \\
& f_1 \arrow[ul] \arrow[rr, dotted] & & f_2 \arrow[dr] & \\
d \arrow[uu, "\mathtt{hb}"] \arrow[ur] & & & & c \arrow[llll, "K"]
\end{tikzcd}
$$

Proof of the simple reduction theorem (2/2)

Finally, consider a cycle with three or more K-edges.

$$
\begin{tikzcd}
a \arrow[rrr, "K"] & & & b \arrow[d, "hb"] \\
d \arrow[u, "hb"] & & & c \arrow[dl, "K"] \\
& \cdot \arrow[ul, "K"] & \cdot \arrow[l, "(\mathtt{hb} \cup K)^+"'] &
\end{tikzcd}
$$

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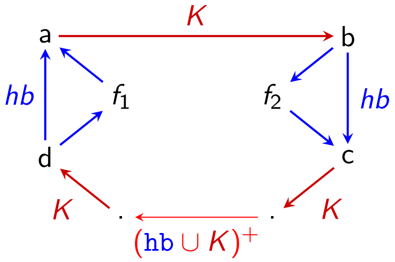

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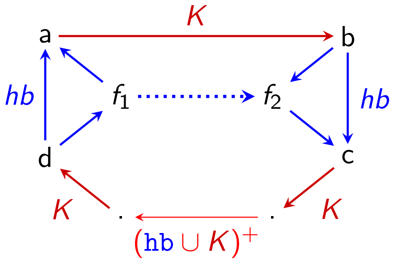

More advanced reduction theorem

Theorem

Let G be a WW-race-free RA-consistent execution, and there exists a set $B \subseteq G.\mathtt{E}$ of protected events such that:

1. $\mathtt{hb} \triangleq (G.\mathtt{po} \cup G.\mathtt{rf})^+$ *is total on* B.
2. *If a races with b in G, then either $a \in B$ or $b \in B$.*
3. *For every G-racy write/update event $a \in B$ and G-racy read event $b \in B$, if $\langle a, b\rangle \in \mathtt{hb}$, then $\langle a, c\rangle, \langle c, b\rangle \in \mathtt{hb}$ for some fence event c.*
4. *For every G-racy write/update event $a \notin B$ and G-racy read event $b \notin B$, if $\langle a, b\rangle \in \mathtt{hb}$, then $\langle a, c\rangle, \langle c, b\rangle \in \mathtt{hb}$ for some fence or protected event c.*

Then, G is SC-consistent.

Note

Because of WW-race-freedom, if $\langle a, b\rangle \in K$, then a is a read and b is a write (or update).

Consider minimal cycle in $(\mathtt{hb} \cup K)$.

- Cycle with two K-edges:

$$
\begin{tikzcd}
\text{a:R} \arrow[rr, "K"] & & \text{b:W} \arrow[d, "\mathtt{hb}"] \\
\text{d:W} \arrow[u, "\mathtt{hb}"] & & \text{c:R} \arrow[ll, "K"]
\end{tikzcd}
$$

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Consider minimal cycle in $(\mathtt{hb} \cup K)$.

- Cycle with two K-edges:

$$\begin{tikzcd}
\in B & \notin B \\
\text{a:R} \arrow[r, "K"] & \text{b:W} \arrow[d, "\mathtt{hb}"] \\
\text{d:W} \arrow[u, "\mathtt{hb}"] & \text{c:R} \arrow[l, "K"]
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d{:}W \arrow[u, "\mathtt{hb}"] & c{:}R \arrow[l, "K"] \\
\notin B & \in B
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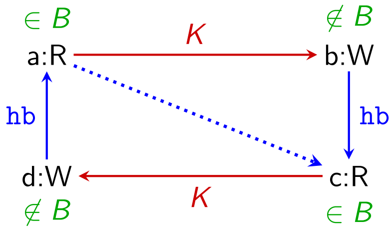

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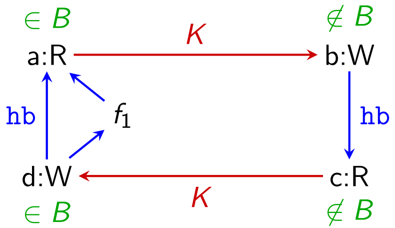

Proof sketch

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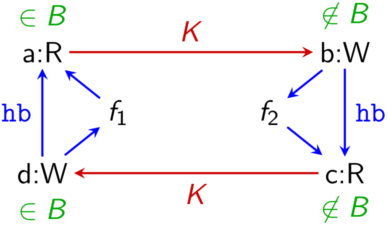

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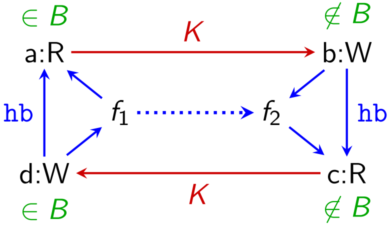

Applying the theorem to RCU

```
rcu_quiescent_state():
   rc[get_my_tid()] := gc; fence();

rcu_thread_offline():
   rc[get_my_tid()] := 0; fence();

rcu_thread_online():
   rc[get_my_tid()] := gc; fence();

synchronize_rcu():
   local was_online := (rc[get_my_tid()] ≠ 0);
   if was_online then rc[get_my_tid()] := 0;
   lock();
     gc := gc + 1;
     fence();
     for i := 1 to N do wait (rc[i] ∈ {0,gc});
   unlock();
   if was_online then rc[get_my_tid()] := gc;
   fence();
```