Information Flow Control for Event Handling and the DOM in Web Browsers

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***Abstract*—Web browsers routinely handle private information. Owing to a lax security model, browsers and JavaScript in particular, are easy targets for leaking sensitive data. Prior work has extensively studied information flow control (IFC) as a mechanism for securing browsers. However, two central aspects of web browsers — the Document Object Model (DOM) and the event handling mechanism — have so far evaded thorough scrutiny in the context of IFC. This paper advances the state-of-the-art in this regard. Based on standard specifications and the code of an actual browser engine, we build formal models of both the DOM (up to Level 3) and the event handling loop of a typical browser, enhance the models with fine-grained taints and checks for IFC, prove our enhancements sound and test our ideas through an instrumentation of WebKit, an in-production browser engine. In doing so, we observe several channels for information leak that arise due to subtleties of the event loop and its interaction with the DOM.**

I. Introduction

A lot of confidential information like passwords, authentication cookies, credit card numbers, search queries and browsing history passes through web browsers. Client-side applications can easily read and leak or misuse this information, due to either malicious intent or insecure programming practices [1], [2], [3], [4]. Browser vendors are sensitive to this problem, but conventional data protection solutions implemented in web browsers have loopholes that can be, and often are, exploited. For example, the standard same-origin policy (SOP) [5], which is intended to restrict cross-domain data flows, can be easily bypassed by malicious programs through cross-domain image download requests that are exempt from the policy by design. This has often been exploited to leak cookies from webpages.

A significant source of security problems in web applications is the lax security model of the ubiquitous client-side programming language JavaScript (JS). In all browsers, third-party JS included in a page runs with the privileges of the page. This enables data leaks when untrustworthy third-party scripts are blindly included by website developers. Content-security policies (CSPs) [6] that allow whitelisting of trusted websites have been implemented to alleviate the problem, but CSPs also disable inline JS and dynamic code insertion (through the JS construct `eval()`), both of which are widely used [7]. More fine-grained data protection methods such as Google's Caja [8], FBJS [9] or AdSafe [10] use static analysis or source rewriting to limit access of third-party code to confidential resources. Although sometimes provably secure [11], these systems restrict third-party code to subsets of HTML and JS to enable analysis.

More generally, all data protection mechanisms discussed above implement some form of one-time *access control* on data available to third-party code. As such, they are completely ineffective when the third-party code legitimately needs confidential data to provide functionality, but must be prevented from disclosing it in unexpected ways. *Information flow control* (IFC) within the web browser is an obvious, holistic method to solve this problem. With IFC, the browser can allow third-party code to access confidential data, but monitor flows of information within the third-party code (either finely or coarsely) and selectively disallow unexpected flows, thus supporting both functionality and security. Unsurprisingly, a number of solutions based on IFC have been proposed for web browsers [12], [13], [14], [15], [16], [17], [18], [19]. However, all these solutions have two significant shortcomings — they either do not handle or abstract over the event handling logic of the browser and they do not handle the browser APIs completely. In this paper, we design, formalize, prove sound and implement an IFC solution that addresses these shortcomings.

Shortcoming 1: Event handling logic. Existing IFC solutions for web browsers do not treat the browser's *event handling* logic completely and accurately. A webpage is reactive: Input events like mouse clicks, key presses and network receives trigger JS functions called *handlers*. The event handling logic of a browser is complex. Each input event can trigger handlers registered not just on the node on which the event occurs (e.g., the button which is clicked), but also on its ancestors in the HTML parse tree. This can leak information implicitly through the presence or absence of links between the node and its ancestors. However, existing work on IFC for web browsers either does not consider the reactive nature of webpages at all (focusing, instead, only on the sequential JS code within a handler) [14], [15], [13], [20], [16] or it abstracts away from the details of the event handling logic, thus side-lining the problem [17], [18].

In contrast, in this work we (a) Demonstrate through working examples that information leaks through the event loop logic are real (and subtle) and, thus, should not be abstracted, (b) Enrich a formal model of the event loop of

a browser with fine-grained IFC to prevent such leaks, (c) Prove that our proposed extension is sound by establishing noninterference, a standard property of IFC, and (d) Implement our IFC solution in WebKit, an in-production browser engine used in many browsers (including Apple's Safari). Our IFC-enriched model of the event loops and the noninterference proof are parametric in the sequential small-step semantics and IFC checks of individual event handlers. Additionally, our solution can be layered over any existing label-based, fine-grained IFC solution for sequential JS that satisfies noninterference, e.g., [14], [15]. To test and evaluate the cost of our IFC checks, we extend Bichhawat *et al.*'s IFC instrumentation for individual handlers in WebKit's JS bytecode interpreter [15].

As a further contribution, we observe empirically that event handlers in web browsers do *not* necessarily execute atomically. *Every* existing work on IFC in web browsers (and beyond) incorrectly assumes the opposite. Chrome, Firefox and Internet Explorer sometimes temporarily suspend an event handler at specific API calls to handle other waiting events. The suspended handlers resume after the other waiting events have been handled. This behavior can be nested. As we show through examples, this kind of preemption can also cause implicit information leaks. We model this preemption in our formalism and our IFC instrumentation and implementation prevent leaks through preemption. This adds complexity to our model: We cannot model the state of the event loop with just one call stack for the current event (as existing work does). Instead, we model the state of the event loop with a stack of call stacks — the topmost call stack is the state of the current event and the remaining call stacks are the states of previously suspended events. We note that in the future, web browsers are expected to aggressively support cooperative yielding and resumption of threads with multiple event loops (this is anticipated by the HTML5 specification [21]); our IFC solution should provide a stepping stone to such general settings.

Shortcoming 2: Browser APIs. Existing IFC solutions for web browsers do not cover all APIs of the DOM specification [22]. The Document Object Model or DOM is the parsed, internal state of a webpage that includes all visible and invisible content, embedded handlers and scripts, JS primitive functions and global browser information. The DOM can be read and modified from JS through many native, standard APIs provided by every browser. These APIs, called the DOM APIs, were introduced into browsers in three stages, now dubbed DOM Levels 1, 2 and 3. The DOM is *the* main shared state (memory or heap) for JS code executing in the browser. Its APIs often have complex implementations and, hence, any IFC solution *should* carefully instrument these APIs for IFC and account for that instrumentation in the soundness proof. However, existing IFC solutions for web browsers either completely ignore the DOM [14], [13], [15] (and consider a JS core with a standard heap), or instrument only a part of the DOM [20], [18], [17], [23], [24]. Other work does not specify how far the DOM was instrumented and does not prevent all leaks [16]. Formal models of the DOM outside of IFC are limited — we know of only two and both are partial [25], [26].

In our work, we model all DOM APIs up to and including Level 3, and instrument them (in WebKit) to track fine-grained taints and enforce IFC. This is nontrivial because we had to consult the implementation of the DOM APIs in WebKit to resolve ambiguities in the standard DOM specification [22]. For instance, in the case of the API `getElementById('id')`, which is supposed to retrieve a unique element `id`, the specification does not specify behavior when several elements have the same `id`. To resolve such ambiguities, we turn to WebKit's implementation. In doing so, we also found a new set of leaks which arise due to optimizations in WebKit. Our model of all DOM APIs can be added to any prior sequential model of JS in the form of extra primitive JS functions. Our noninterference proof (for the event loop) also carefully analyzes our IFC instrumentation of every DOM API and shows that our design prevents information leaks. Our model of the DOM, which may be of interest even outside of IFC, is formalized as (type-checked) OCaml code and is available online from the authors' homepages. Due to lack of space, we do not describe the DOM API or our instrumentation of it in any detail in this paper, except through examples (we focus on the conceptually harder event loop in the technical sections of this paper).

Summary of contributions. To the best of our knowledge, this is the first web browser IFC work that handles event loops and the DOM up to Level 3. To summarize, we make the following contributions.

- We formalize the event handling loop of a browser, highlighting how it can leak secrets implicitly, and develop a fine-grained dynamic IFC monitor for it.
- We develop a formal model of the DOM up to Level 3. The model is abstracted from the DOM specification and its implementation in an actual browser engine (WebKit). We enrich our model with provisions for fine-grained IFC.
- We prove a form of reactive noninterference for a termination-insensitive attacker. Our proofs are parametric on preemption points and a provably sound IFC monitor for sequential JavaScript.
- We implement these concepts in a fully-functional browser (Apple's Safari, based on WebKit) and observe moderate performance.

```
1 l = false, t = false
2 if (h == false)
3   t = true
4 if (t != true)
5   l = true
```

Listing 1: Example for implicit flow

II. Background

A. Information Flow Control

Information flow control (IFC) refers to controlling the flow of (confidential) information through a program based on a given security policy. Typically, pieces of information are classified into security labels and the policy is a lattice over labels. Information is only allowed to flow up the lattice. For illustration purposes often the smallest non-trivial lattice $L < H$ is used, which specifies that public (low, L) data must not be influenced by confidential (high, H) data. In our instrumentation labels are drawn from a product lattice where each dimension represents a unique web domain. IFC can be used to provide confidentiality (or integrity) of secret (trusted) information. We are only interested in confidentiality here.

In general, information can flow along many channels. Here, we consider *explicit* and *implicit* flows. Covert channels like timing or resource usage are beyond the scope of this work. An explicit flow occurs as a result of direct assignment, e.g., the statement `public = secret + 1` causes an explicit flow from `secret` to `public`. An implicit flow occurs due to the control structure of the program. For instance, in the program `public = false; if (secret) public = true`, the final value of `public` is equal to the value of `secret` even though there is no direct assignment mentioning both `secret` and `public`. Leaking a bit like this can be magnified into leaking a bigger secret bit-by-bit [27].

Research has considered static methods such as type checking and program analysis, which verify the security policy at compile time [28], [29], [30], [31], dynamic methods such as black-box approaches as well as attaching secrecy labels to runtime values and tracking them through program execution [32], [33], [14], [34], [35], [36], and hybrid approaches that combine both static and dynamic analyses to add precision to the analysis [37], [38], [39], [15] for handling the leaks described above. The correctness of these approaches is often stated in terms of a well-defined property known as *noninterference* [40], which basically stipulates that high input of a program must not influence its low output. While noninterference is too strong a property in practice, it is a useful soundness check for IFC mechanisms.

We are interested in IFC through runtime monitoring with labels attached to all values. Preventing explicit flows that violate noninterference is trivial via runtime monitoring, once all values in the system are labeled. However, it can be difficult to prevent leaks due to implicit flows. Dynamic IFC approaches usually use a notion of context label ($\mathcal{PC}$), which represents an upper bound on the labels of all the values that have influenced the control flow at the current instruction, and join this label with the label derived from explicit flow for every variable assignment. However, it can be shown that this is not sufficient for noninterference [41] when labels attached to variables may change over time, as even assignments in code that is *not* executed may lead to implicit flows. Listing 1 illustrates unexecuted branches that may leak information. This code snippet effectively copies the (secret) value of `h` into the public variable `l` via another public variable `t` without `l` being labeled secret. This is because *either* the first condition is true or the second but never both. The *no-sensitive-upgrade* (NSU) check [42], [32] rejects such programs by prohibiting modification of a public variable in a secret context (when the $\mathcal{PC}$ label is high), terminating the program if it tries to do so. In this program, when `h` is `false`, the assignment on line 3 is forbidden and the program is terminated. Programs executed under NSU satisfy the soundness property *termination-insensitive non-interference* [29]. Intuitively, this soundness criterion requires the absence of information leaks for an attacker who cannot observe termination of programs (a formal definition is given in Section IV).

B. Document Object Model and Event Handling

Document Object Model: The *document object model* (DOM) is the parsed, internal state of a webpage that includes all visible and invisible content, embedded handlers and scripts, JS primitive functions and global browser information. It can be accessed from JS programs via the DOM API, which provides interfaces for JS programs to read, modify, create and delete parts of its state. DOM API calls have been added to browsers gradually in stages that are dubbed levels. The current standard implemented in most browsers is Level 3 (which subsumes Levels 1 and 2). The DOM graph, which represents the visual content described by HTML, can be navigated in various directions using API calls, e.g., from a node to its parent, to its first and last children, to its left and right siblings, etc.

Nodes of the DOM graph have various types, like an element node, a text node, a document fragment and many others. All of these are well-defined data structures in the DOM specification [22]. A special kind of data structure of significance to us is the *live collection*. It is returned by some DOM search APIs. A live collection always represents the current state of the DOM graph, i.e., changes in the DOM graph are reflected in future uses of these collections. As an example, consider the function call `document.getElementsByTagName('div')`. This calls returns a reference to a list containing all elements (element nodes) that have the tag name `div`. If another node with tag name `div` is added to the DOM graph after the call,

it will be present in subsequent uses of the list. Similarly, if an element with tag name `div` is removed from the DOM graph, this element is removed from the list automatically.

Event handling: Web pages may be reactive. They can respond to *events* like mouse clicks, network responses and key presses by invoking *event handlers*, which are JS functions. Every event has a *target*, a node in the DOM graph, where the event originates (e.g., if the mouse is clicked on a button, then the button would be the target of the resulting `click` event). Event handlers for specific events can be associated with every node programmatically through the browser API `node.addEventListener(event, handler, boolean)`. Every handler is registered with one of the following attributes: *target and bubble* or *capture and target* by setting the `boolean` third argument to `false` and `true`, respectively. The meaning of these attributes is explained below.

The event loop of a browser is complex. Browsers maintain a list of incoming, pending events called the *event queue*. Events in the queue are processed one at a time (not necessarily in FIFO order). The processing of an event is called a *dispatch*. When an event is dispatched, the handlers registered for the event associated with the event's target node are executed. Additionally, certain handlers registered for the event associated with the nodes on the entire path from the target to the root of the DOM graph are also executed. This path is called the *propagation path*. To dispatch an event, first, this propagation path is computed by traversing the DOM graph. This path remains fixed during event dispatch even though the shape of the DOM graph may change due to the execution of handlers. Next, the handlers are executed in three phases:

- The *capture phase* executes all capture and target handlers associated with all nodes from the root to the target's parent, starting from the root.
- The *target phase* executes all the handlers associated with the target.
- The *bubble phase* executes all target and bubble handlers associated with all nodes from the target's parent to the root, starting from the target's parent.

Finally, the browser may execute *default actions* (the browser's in-built actions) associated with the event. For example, middle-clicking a url in Chrome opens the url in a new tab.

Events can be dispatched by external actions (like a physical mouse click) or programmatically using the API call `dispatchEvent()`. When an event is dispatched programmatically, default actions are usually not executed. An exception to this is the `click` event.

Every dispatched event has three flags which can be set by any executing event handler through API calls to modify the execution of subsequent handlers. The flag `stopImmediatePropagation` terminates handling of the event immediately after the current handler ends. No other handlers are executed, but default actions are executed. The flag `stopPropagation` is similar but it terminates handling after all handlers associated with the current node have executed. The flag `defaultPrevented` prevents the execution of default actions.

```
1  var p = document.getElementById('para');
2  p.onclick = function() {
3     alert('In click');
4     p.innerHTML += 'click';
5  };
6  window.onresize = function() {
7     p.innerHTML = 'resize ';
8  };
```

Listing 2: Preemption in browsers

Handler preemption: An event handler can be paused or suspended at specific API calls like `alert()` or `confirm()` that wait for user response. When suspended at these APIs, some browsers choose to dispatch other events in the event queue, which makes the execution of JS in these browsers resemble cooperative scheduling. Consider the snippet in Listing 2. Assume that the page has a paragraph element with id `para`, which is bound to the variable `p`. The script registers two handlers: one for the `onclick` (click) event on `p` and the other for the `onresize` (resize) event on the global object, `window`. The user clicks the paragraph `p`, which displays an `alert` dialog box. Before dismissing the dialog box, the user resizes the main window thereby registering an `onresize` event. In some browsers, the `onresize` handler will execute while the `onclick` handler is still suspended. This will cause the word `resize` to appear in the paragraph `p` before the word `click`. (We verified this behavior on Chrome version 40.0.2214.111.) On other browsers, resizing the window will not be allowed until the alert dialog box is dismissed or the `onresize` event will not be dispatched until the dialog box is dismissed and `onclick` has finished execution. To account for this browser-dependent behavior, we parametrize our model with a set of preemption points — the API calls at which handler execution may be preempted.

III. Overview of Challenges and Approach

In this section, we highlight some possible information leaks in client-side JS due to subtleties of handler preemption, the event loop logic, the DOM and browser optimizations, and explain at a high-level how our work prevents these leaks. But, first, we explain our attacker model.

Approach and attacker model. We perform fine-grained, flow-sensitive taint tracking in the DOM (and through the event loop), so we assume here onward that taints are attached to individual fields of DOM elements. Thus, the parent, first and last child, and left and right sibling pointers

```
1   function foo() {
2     ...
3     pub = true;
4     if (sec)
5       preemption-point
6     ...
7     pub = false;
8   }
9
10  function bar() {
11    conf = pub;
12  }
```

Listing 3: Implicit leak via preemption

of a DOM node can have independent taints and the content of the node can have yet another taint. This is quite standard in fine-grained dynamic taint tracking [14], [15]. Conceptually, taints may be drawn from an arbitrary security lattice; our prototype implementation uses a subset lattice where the taint on a field is an upper bound on the set of web domains which may have influenced the field. We also attach labels to individual events and event handlers. A value v labeled with label ℓ is written v^ℓ.

Our attacker resides at an arbitrary security level $\mathcal{A}$ of the lattice. We assume that the attacker can observe all references and values $\ell \leq \mathcal{A}$ that are reachable from the global object (`window` object) of the browser. The attacker cannot directly observe internal data structures like the call-stack, event queues, etc. We limit attention to a termination-insensitive attacker. Leaks due to termination, timing, progress and other side channels are outside our threat model.

Examples Through a series of examples, we demonstrate subtle implicit leaks through handler preemption, event loops, the DOM APIs and browser optimizations. These pose a challenge for building a dynamic flow-sensitive IFC monitor. For simplicity, our examples demonstrate only one-bit leaks. In all our examples, we assume the lattice $L < H$ (low $<$ high) and an attacker at level L.

Handler preemption: Our first example highlights a possible leak due to preemption in the middle of a handler. As noted earlier, every prior work on IFC for web browsers (and reactive systems in general) ignores such preemption and, hence, will miss this attack. Listing 3 describes two handlers, `foo` and `bar`, for two arbitrary events e_1 and e_2, respectively. We assume that both events are in the event queue already and that the handler for e_1, i.e., `foo` is currently executing, while the other handler has not yet started executing. In the function `foo`, we initialize a variable `pub` to `true` and branch on a H-labelled variable `sec` to decide whether or not to execute an instruction at line 5, which is a preemption point according to the browser semantics. If `sec` is `true`, then at line 5, `foo` will be suspended and `bar` will start executing while `pub` is `true`. From the code of `bar`, it is clear that the variable `conf` will end up with the value `true`. If, on the other hand, `sec` is `false`, then the preemption point is never executed, so `bar` executes after `foo` sets `pub` to `false` and `conf` ends with `false`. Hence, in a browser that preempts at line 5, this program implicitly copies `sec` to `conf`.

The relevant question is how we may prevent this leak with dynamic IFC. A naive solution, based on handling of implicit flows in sequential programs, would be to carry the $\mathcal{PC}$ from `foo` to `bar` at line 5, i.e., to execute `bar` with $\mathcal{PC} = H$ if line 5 executes. Although intuitive, this naive solution is *unsound*: It prevents the leak only if `conf` is initially labeled L, but still allows the leak if `conf` is initially labeled H. If `conf` is initially labeled L, when `sec` is $\texttt{true}^H$, `bar` executes with $\mathcal{PC} = H$ and this prevents the assignment $\texttt{conf}^L = \texttt{pub}$ due to the no-sensitive-upgrade (NSU) check. Hence, the program does not leak (to a termination-insensitive attacker). However, if `conf` is initially labeled H, this assignment is allowed, so the program leaks information. More specifically, if `conf` is initially labeled H, then `conf` ends with value $\texttt{true}^H$ if `sec` is $\texttt{true}^H$, and with value $\texttt{false}^L$ if `sec` is $\texttt{false}^H$. The final values of `conf` — $\texttt{true}^H$ and $\texttt{false}^L$ — are distinguishable for the attacker. So, $\texttt{sec}^H$ is leaked by the program.

To solve this problem, we attach minimum $\mathcal{PC}$ labels to all handlers and execute a handler at a preemption point only when the handler's $\mathcal{PC}$ label is at least as high as the $\mathcal{PC}$ of the context. In this example, `bar` must have a label at least H in order to execute after line 5. This minimum label ensures that after `bar` executes, `conf` has label H irrespective of earlier code paths so, trivially, there are no information leaks. Note that we do *not* specify labels for handlers through additional program annotations. Instead, a handler inherits the label of the context in which the handler was associated with the event. This is essential for soundness, else the handler's execution may implicitly leak information from the context in which the handler was associated with the event.

Event phases: As described in Section II, event handling happens in several phases and the handlers executed in response to an event depend on the shape of the DOM, which is itself dynamic. This can result in implicit information leaks unless the IFC monitor is carefully designed. The example in Listing 4 illustrates this point. We have a low variable `pub` initially set to `false`, and two nodes `a` and `b`. The `onClick` handlers (the handlers invoked when the mouse is clicked on the node) for `a` and `b` are set to `aCk` and `bCk`, respectively. `aCk` is a capture and target phase handler (third argument to `addEventListener()` is `true` on line 2) and `bCk` is a target and bubble phase handler (third argument to `addEventListener()` is `false` on line 3). Based on a high variable `sec`, we either make `b` a child of `a` or we do not. Then, on line 6, a mouse click is

```
1  pub = false
2  a.addEventListener ("click", aCk, true)
3  b.addEventListener ("click", bCk, false)
4  if (sec)
5    a.appendChild (b)
6  b.dispatchEvent ("click")
7
8  function aCk () {
9    pub = true
10 }
11 function bCk () { ... }
```

Listing 4: Example leak due to lack of *propagation path labels* across event phases

programmatically dispatched on `b`. `b` is the target of the event, so `bCk` will definitely execute. However, if `sec` is `true`, then `aCk` will also execute (before `bCk`) because `aCk` is registered as a capture phase handler and `a` is the parent of `b`. The execution of `aCk` will change `pub` to `true`. Hence, `sec` is copied to `pub` implicitly.

To prevent this leak, let us examine why the leak happens. Essentially, depending on the value of `sec`, either there exists a link between `a` and `b` or there does not. This link then determines whether or not `aCk` executes. Hence, the leak can be prevented by ensuring two things: (1) The link between `a` and `b` is labeled with the $\mathcal{PC}$ when the link is formed (here, the label would be H) and, (2) When a handler tied to a node such as `aCk` is executed, the execution's $\mathcal{PC}$ is at least as high as the labels of all links on the path from the target node to this node (in this case, the link from `a` to `b`, which would be labelled H). In our DOM instrumentation, (1) happens for free because links between nodes are ordinary pointer fields, and all fields inherit the $\mathcal{PC}$ label of the context upon update. On the other hand, (2) requires special care: Our instrumentation executes all handlers for an event with a $\mathcal{PC}$ at least as high as the *propagation path label*, which is the join of the labels on the pointers in the propagation path of the event's target. This subsumes (2) and also prevents similar implicit leaks.

Live collections: Some DOM API functions that search the DOM graph return a live collection of nodes. A live collection is automatically updated as nodes that match the search criteria are added or removed from the DOM graph. This can leak information implicitly. Listing 5 shows a simple example. The variable `nodes` is bound to the live collection of all nodes in the DOM that have the tag `div`. The length (size) of this collection is snapshotted in the variable `x`. Depending on the value of a high variable `sec`, either a new node `newNode` with tag `div` is added to the DOM or it is not. If the node is added, this will automatically (and implicitly) update the live collection. The new length of the live collection is snapshotted in `y`. If `y` and `x` are different, then `sec` is `true`, else it is `false`. This is an implicit leak of `sec` to `x` and `y`.

```
1  nodes =
2     document.getElementsByTagName("div")
3  x = nodes.length
4  newNode = document.createElement("div")
5  if (sec)
6    document.body.appendChild(newNode)
7  y = nodes.length
```

Listing 5: Example leak via live collections

Two recent proposals [24], [20] offer similar solutions to this problem: They require the programmer to provide a security label for every tag like `div`. The IFC monitor ensures that this label is an upper bound on the $\mathcal{PC}$ of the context in which any node with that tag is added to the DOM. Any value computed from the live data structure, e.g., `length`, inherits this label. In our example, the tag `div` must have the label high in order for line 7 to execute. So, `nodes.length`, `x` and `y` would all be labelled high, preventing the leak.

Although elegant, this solution requires the programmer to specify labels for each tag. Our approach, on the other hand, does not require any assistance from the programmer. We rely on the observation that browsers implement properties of live data structures like `length` by traversing the DOM graph.[1] Consequently, simply applying our usual IFC checks to the implementations of methods of live data structures yields sound enforcement. In our example, if line 6 succeeds, then due to the NSU check on pointer update, the *last-child* pointer of `document.body` must already be labeled high. Hence, irrespective of whether or not `newNode` is added, traversing the DOM graph to compute the `length` field will always return a high value. Hence, `x` and `y` will be labelled high, preventing any leak. Note that our solution requires fine-grained labels on all fields in the DOM (which we have). Prior work mentioned above did not include a full instrumentation of the DOM and, hence, could not have adopted our solution.

Browser optimizations: Browsers often create auxiliary internal data structures to speed up commonly invoked DOM API functions. Any IFC instrumentation of the DOM must also label these auxiliary data structures. This can be subtle as we illustrate here. WebKit, the browser engine we instrument, maintains a HashMap from node ids to DOM nodes to speed up the very common lookup function `getElementById('iden')`, which returns the DOM node with id `iden`. Whenever a node is inserted into the DOM (as the child of another already present node), the node is also added to this HashMap. However, for various reasons, this map contains a *subset* of all nodes in the DOM. If a certain id does not exist in the HashMap, then the DOM

[1] A live data structure is marked invalid as soon as there is any change to the DOM. A subsequent method call on the data structure results in a fresh DOM traversal.

```
1  c.setAttribute("id","ifc")
2  if (sec)
3    b.appendChild(c)
4  a.appendChild(b)
5  x = document.getElementById("ifc")
```

Listing 6: Example leak showing insufficiency of $\mathcal{PC}$ for preventing leak via getElementById

must be traversed to search. Clearly, to prevent an implicit leak, it is essential that the $\mathcal{PC}$ of the context in which a node is added to the DOM also labels the entry of the node in the HashMap (if the entry exists). However, it turns out that just this labeling is *insufficient* in cases where the node being added to the DOM has children.

As an illustration, consider the example of Listing 6. Three nodes `a`, `b` and `c` have been created before the program starts, but only `a` is already attached to the DOM. Assume that `b` is a high node, i.e., its own label and its fields are all labeled H. The program assigns `c` the id `ifc`, which will be searched for later. Based on the value of a high variable `sec`, `c` is either made a child of `b` or not. Then, `b` is made a child of `a`. This adds `b` and possibly `c` to the DOM. Since this instruction executes with $\mathcal{PC} = L$, following the labeling rule described above, `c`'s entry in the HashMap, if any, will be labeled L. The last line of the program searches for the id `ifc` in the DOM and stores the result in variable `x`. At the end of the program, `x` will be $\texttt{null}^H$ if `sec` is $\texttt{false}^H$ (note that the DOM traversal to discover that a node with id `ifc` does not exist will pass through `b`, which is labelled H and, hence, will return $\texttt{null}^H$). On the other hand, `x` will be $\texttt{c}^L$ if `sec` is $\texttt{true}^H$ due to a successful HashMap lookup. Since $\texttt{null}^H$ and $\texttt{c}^L$ are distinguishable for the adversary, this program leaks `sec` into `x`.

This leak happens because, when `sec` is `true`, we completely ignore the H label of the link between `b` and `c` when labeling `c` in the HashMap. The correct rule for labeling the HashMap on insertion is that when a node, say `b`, is added to the DOM graph, any node accessible from `b` must be added to the HashMap with a label that is equal to the *join of all labels* on the path from `b` to that node. With this rule in place, when `c` is inserted into the HashMap, its label is H, not L, and this prevents the leak.

IV. Information Flow Control for the DOM and the Event Loop

Our central technical contribution is IFC-enhanced models of the DOM Core Level 3 [22] and of the event loop of a web browser, including event preemption. Our models plugs into any model of sequential JS execution within a handler if the sequential model uses dynamic fine-grained taints for IFC. Our model of the DOM extends such a sequential model with additional primitive functions (the DOM API) and our model of event loops provides a wrapper

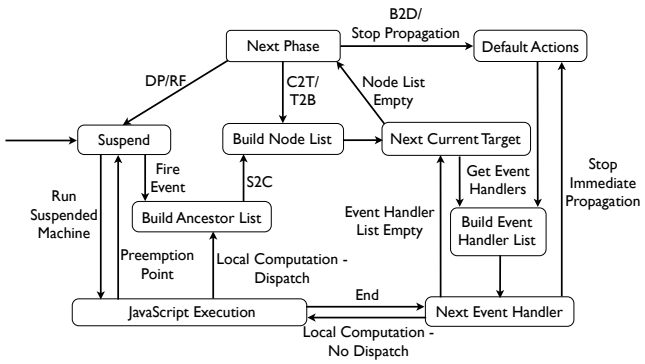

Figure 1: State machine description of an event loop

around such a model, resulting in a reactive system. To prove noninterference, we assume that the IFC on sequential JS is noninterfering and satisfies some specific lemmas (like the standard confinement lemma). The specific sequential JS model we use is taken from [15].

IFC for the DOM. As described in Section II-B and illustrated in Section III, the DOM is a graph whose nodes correspond to various elements of a web page. Technically, each node is a JS object. The standard rules of IFC from the sequential JS apply to these objects as well. In particular, every field of every node, including every pointer to the node's children, siblings and parent, is given a separate security label. This label is always at least as high as the $\mathcal{PC}$ of the context in which the field was last modified.

Separately, we build an OCaml model of all DOM API functions. We follow the DOM specification [22] and refer to its implementation in WebKit (our target browser engine) to resolve ambiguities and to add additional data structures like the HashMap of Section III(d), which are not part of the specification. Finally, we instrument the API functions with IFC checks. We follow the NSU rule to prevent implicit flows. Two interesting aspects of the model were highlighted in Section III(c) and (d). We do not describe this model in any further detail here. Instead, in the rest of this section, we focus on our model of a browser's event loop and its IFC enhancement.

A. The event loop and its IFC enhancement

As explained in Section II, the event loop is a transition system that collects events and dispatches them to handlers through a reasonably complex logic. Additionally, event handlers can be preempted at browser-specific API calls. Fig. 1 describes the event loop as an abstract state machine. The machine starts in a suspended state (and returns to it when there are no events; this transition is not shown). The occurrence of an event (called "Fire event") transitions the machine to what we call the *start phase*. In this phase, the propagation path, or the list of nodes from the target of the event to the root of the DOM graph, is computed. This list

is fixed for the duration of the event's dispatch. Then, the machine transitions through the capture, target and bubble phases described in Section II. In Fig. 1, the transitions are labelled S2C (start to capture), C2T (capture to target), T2B (target to bubble), B2D (bubble to default). The last phase of execution is what we call the default phase, where the default actions are executed.[2] The execution of some handlers can be bypassed using the flags `stopPropagation` and `stopImmediatePropagation`. When either the default actions are prevented (transition DP), an event has been handled completely (transition RF), or a preemption point is reached, the machine enters the suspend state, where it can dispatch other events.

Concretely, we model a state of the machine as a pair $\kappa = \langle \Sigma, \theta \rangle$, where Σ is a stack of frames and θ is a shared heap (which includes the DOM graph). Each frame corresponds to one event which has been dispatched, but whose handling has not yet completed. The top most frame corresponds to the event currently being handled; every other frame has been suspended (prior to completion) to handle events in frames above it. The stack frame is a rich data structure that encodes the entire state of an event's dispatch including the state of the currently running handler, phase information and handlers that have not yet run. The stack frame is a tuple $\nu = \langle C, M, \ell, E, N, \mathcal{A}, \mathcal{N}, \mathcal{H}, \mathcal{P} \rangle$, where:

- C : While a handler is active, C is a configuration of the sequential JS machine, minus the heap, which is shared. It has the form $C = \langle \iota, \sigma, \rho \rangle$, where ι is a pointer to the instruction (in the heap) to be executed next, σ is the call stack of the current handler, and ρ is the $\mathcal{PC}$-stack for the current handler. We write $\iota(C)$, $\sigma(C)$, and $\rho(C)$ to denote the current instruction, call-stack and $\mathcal{PC}$-stack of the configuration C. Between two phases or between two handlers, when no handler is active, C temporarily takes the form ϕ.
- M : Either S (suspended) or R (running). All frames in Σ except the topmost are always in the S state. When the topmost frame is in state S, a new event can be dispatched by adding a new frame on top.
- ℓ : The security label in which the frame ν was created. This indicates the minimum level at which instructions execute in that frame. The function $\Gamma(\nu)$ returns the value of ℓ in frame ν.
- E : A pointer to the event object for which the frame ν was created.
- N : A pointer to the target object for the event E.
- $\mathcal{A}$: A list of pointers to nodes in the propagation path of the target. This list starts with the target's parent node and ends at the root of the DOM. The list is fixed when the frame is created and does not change afterward.
- $\mathcal{N}$: A list of pointers to nodes on which handlers have yet to be run in the current phase. This list is re-populated at every phase transition and gets smaller as the phase progresses.
- $\mathcal{H}$: A list of pointers to event handlers (functions) at the current node that are yet to be executed.
- $\mathcal{P}$: The current phase of execution for the event. It can be $\mathcal{S}$, $\mathcal{C}$, $\mathcal{T}$, $\mathcal{B}$ or $\mathcal{D}$ for start phase, capture phase, target phase, bubble phase and default phase, respectively.

The transitions of this state machine are described as small step operational semantics in Figure 2. The transition relation has the form $\langle \Sigma, \theta \rangle \twoheadrightarrow_\alpha \langle \Sigma', \theta' \rangle$ where $\alpha = \cdot | (e, o)$ is a label. $\alpha = (e, o)$ labels the firing of event e on target object o. All other transitions are labeled $\alpha = \cdot$. The top frame of a stack S is denoted $!S$. Some of the rules make use of meta functions like `getPathLabel`, which we describe informally as we explain the rules; all of these functions have a formal description in our OCaml model. We describe the rules of Figure 2 below.

`Local Computation-No Dispatch`: When the current instruction in the top frame of Σ (i.e., $\iota(!\Sigma.C)$) is neither *dispatchEvent* (programmatic event dispatch) nor a preemption point, then we can execute this instruction using the small-step semantics of sequential JS. The relation $\theta, C \rightarrow \theta', C'$ is the small-step transition relation of the underlying sequential JS machine.

`Local Computation-Dispatch`: This rule executes the programmatic dispatch call, *dispatchEvent*. We push a new frame on Σ. The current frame's state changes to S (suspended). The function `getPathLabel` joins the labels of the nodes on the propagation path of the event target `o`. The function `getAncestors` returns the propagation path of `o`. We set the label ℓ in the new top frame to the join of the current $\mathcal{PC}$ and the path label, as explained in Section III(b). The current phase of the new frame is set to the start phase, $\mathcal{S}$.

`Preemption-Point`: If the current instruction is a preemption point according to the browser semantics (captured in the browser-dependent check $isPreemptionPoint(\iota(C_m)) = true$), then this rule changes the state of the topmost frame of Σ to S. This allows another event to fire through the rule `Fire Event` (which applies only when the topmost frame has state S).

`Run Suspended Machine`: If the frame on the top of the stack Σ is currently suspended, then it may be resumed by setting the state to R.

`End`: When the current handler has finished executing ($\iota(C_m) = end$), this rule sets the configuration C in the top frame to ϕ. This allows the machine to transition phase or execute the next handler.

`Fire Event`: This rule fires a queued (non-programmatic) event `e` on the target node `o`. The rule is mostly similar to `Local Computation-Dispatch`, but additionally prevents the firing of a low event in a high

[2]The words "start phase" and "default phase" are not taken verbatim from the event specification [43]. We use these terms as they are similar to capture phase, target phase and bubble phase, which the specification mentions.

Local Computation-No Dispatch

$$\frac{!\Sigma.C = C_m \quad !\Sigma.M = R \quad \theta, C_m \to \theta', C'_m \quad isPreemptionPoint(\iota(C_m)) = false \quad \iota(C_m) \neq o.dispatchEvent(e) \quad \Sigma' := \Sigma[!\Sigma.C := C'_m]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta'}$$

Local Computation-Dispatch

$$\frac{!\Sigma.C = C_m \quad !\Sigma.M = R \quad \theta, C_m \to \theta, C'_m \quad \iota(C_m) = o.dispatchEvent(e) \quad \Sigma'' := \Sigma[!\Sigma.C := C'_m, !\Sigma.M := S] \quad e.eventType \neq \text{“}\texttt{click}\text{”} \implies e.defaultPrevented := true \quad \ell_p = getPathLabel(o, \theta) \quad \mathcal{A} = getAncestors(o, \theta) \quad \ell := \Gamma(!\rho(C_m)) \sqcup \ell_p \quad \Sigma' := \langle \phi, S, \ell, e, o, \mathcal{A}, [], [], \mathcal{S} \rangle :: \Sigma''}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Preemption-Point

$$\frac{!\Sigma.C = C_m \quad !\Sigma.M = R \quad \theta, C_m \to \theta', C'_m \quad isPreemptionPoint(\iota(C_m)) = true \quad \Sigma' := \Sigma[!\Sigma.C := C'_m, !\Sigma.M := S]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta'}$$

Run Suspended Machine

$$\frac{!\Sigma.C = C_m \quad !\Sigma.M = S \quad \Sigma' := \Sigma[!\Sigma.M = R]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

End

$$\frac{!\Sigma.C = C_m \quad !\Sigma.M = R \quad \iota(C_m) = end \quad \Sigma' := \Sigma[!\Sigma.C := \phi]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Fire Event

$$\frac{!\Sigma.M = S \quad \ell_p = getPathLabel(o, \theta) \quad \mathcal{A} = getAncestors(o, \theta) \quad \ell := \ell_p \sqcup \Gamma(\theta(e)) \sqcup \Gamma(\theta(o)) \quad \Big(!\Sigma.C = \phi \wedge \ell \geq \Gamma(!\Sigma)\Big) \vee \Big(\ell \geq \Gamma(!\rho(!\Sigma.C))\Big) \quad \Sigma' := \langle \phi, S, \ell, e, o, \mathcal{A}, [], [], \mathcal{S} \rangle :: \Sigma}{\Sigma, \theta \twoheadrightarrow_{(e,o)} \Sigma', \theta}$$

Start-to-Capture

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{P} = \mathcal{S} \quad nl = reverse(!\Sigma.\mathcal{A}) \quad \Sigma' := \Sigma[!\Sigma.\mathcal{N} := nl, !\Sigma.\mathcal{P} = \mathcal{C}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Capture-to-Target

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{P} = \mathcal{C} \quad !\Sigma.\mathcal{N} = [] \quad !\Sigma.\mathcal{H} = [] \quad \Sigma' := \Sigma[!\Sigma.\mathcal{N} := [!\Sigma.N], !\Sigma.\mathcal{P} = \mathcal{T}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Target-to-Bubble

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{P} = \mathcal{T} \quad !\Sigma.\mathcal{N} = [] \quad !\Sigma.\mathcal{H} = [] \quad !\Sigma.E.bubbles = true \implies !\Sigma.\mathcal{N} := !\Sigma.\mathcal{A} \quad \Sigma' := \Sigma[!\Sigma.\mathcal{P} = \mathcal{B}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Bubble-to-Default

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{P} = \mathcal{B} \quad !\Sigma.\mathcal{N} = [] \quad !\Sigma.\mathcal{H} = [] \quad !\Sigma.E.defaultPrevented = false \quad nl = getDefaults(!\Sigma.\mathcal{A}, !\Sigma.N, !\Sigma.E.bubbles) \quad \Sigma' := \Sigma[!\Sigma.\mathcal{N} := nl, !\Sigma.\mathcal{P} = \mathcal{D}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Default Prevented

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{P} = \mathcal{B} \quad !\Sigma.\mathcal{N} = [] \quad !\Sigma.\mathcal{H} = [] \quad !\Sigma.E.defaultPrevented = true \quad \Sigma' := \Sigma[!\Sigma.\mathcal{P} = \mathcal{D}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Remove Frame

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{P} = \mathcal{D} \quad !\Sigma.\mathcal{N} = [] \quad !\Sigma.\mathcal{H} = [] \quad \Sigma' := \Sigma - !\Sigma}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Get Event Handlers

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{N} = n :: \mathcal{N}' \quad !\Sigma.\mathcal{H} = [] \quad !\Sigma.E.stopPropagation = false \quad ehl := getEventHandlers(!\Sigma.E, n, !\Sigma.\mathcal{P}, \theta) \quad \Sigma' := \Sigma[!\Sigma.\mathcal{N} := \mathcal{N}', !\Sigma.\mathcal{H} := ehl]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Stop Propagation

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{H} = [] \quad !\Sigma.E.stopPropagation = true \quad \Sigma' := \Sigma[!\Sigma.\mathcal{N} := [], !\Sigma.\mathcal{P} := \mathcal{B}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Run Event Handlers

$$\frac{!\Sigma.C = \phi \quad !\Sigma.\mathcal{H} = (\ell_c, f_m) :: \mathcal{H}' \quad !\Sigma.E.stopImmediatePropagation = false \quad \rho(C_m) := [!\Sigma.\ell \sqcup \ell_c \sqcup \Gamma(f_m)] \quad \iota(C_m) := null \quad \sigma(C_m) = emptyCallFrame \quad \Sigma' := \Sigma[!\Sigma.C := C_m, !\Sigma.\mathcal{H} := \mathcal{H}']}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Stop Immediate Propagation

$$\frac{!\Sigma.C = \phi \quad !\Sigma.E.stopImmediatePropagation = true \quad \Sigma' := \Sigma[!\Sigma.\mathcal{N} := [], !\Sigma.\mathcal{H} := [], !\Sigma.\mathcal{P} := \mathcal{B}]}{\Sigma, \theta \twoheadrightarrow \Sigma', \theta}$$

Figure 2: Semantics of event handling

context, to handle the problem described in Section III(a).

`Start-to-Capture`: This rule transitions from the start to the capture phase. The pending node list $\mathcal{N}$ is initialized to nodes which must be traversed during the capture phase, which is the reverse of the propagation path, $\mathcal{A}$.

`Capture-to-Target`: If the current phase is capture ($\mathcal{C}$) and it has ended, i.e., there is currently no executing handler ($C_m = \phi$), and the node list $\mathcal{N}$ and the event handler list $\mathcal{H}$ are both empty, then this rule shifts the machine to the target phase. In the target phase, only the target node (N) has to be processed, so the pending node list $\mathcal{N}$ is initialized to $[!\Sigma.N]$.

`Target-to-Bubble`: This is similar to `Capture-to-Target` but transitions from the target phase to the bubble phase. In the bubble phase, all nodes on the propagation path $\mathcal{A}$ must be processed, so the pending node list $\mathcal{N}$ is initialized to $\mathcal{A}$.

`Bubble-to-Default`: Once the bubble phase ends, we transfer to the default phase, but only if the event E's flag `defaultPrevented` is unset. In the default phase, default actions will be run on all nodes provided by the browser-specific function $getDefaults()$, so we initialize the pending node list $\mathcal{N}$ to the output of this function.[3]

`Default Prevented`: If at the end of the bubble phase E.`defaultPrevented` is set, then we skip the default phase. We set the phase to $\mathcal{D}$ and the pending node list and the pending handler list to empty. This emulates the end of the default phase.

`Remove Frame`: If there is no executing handler ($C_m = \phi$), the phase is set to $\mathcal{D}$ (default), and the pending node list $\mathcal{N}$ and the pending event handler list $\mathcal{H}$ are both empty, then the current event has been fully handled. So, the top (current) frame of the stack ($!\Sigma$) is removed.

`Get Event Handlers`: If no handler is currently executing ($C_m = \phi$), and the pending handler list $\mathcal{H}$ is empty, then all the handlers in the current node have been processed. So, we obtain the handlers for the next node in the pending node list $\mathcal{N}$ using the function `getEventHandlers`. This function takes as parameters the event, the (next) node, the phase and the heap and returns a list of event handlers. This rule applies only if the `stopPropagation` flag of the event is unset.

`Stop Propagation`: In the same starting state as the previous rule, if the `stopPropagation` flag of the event is set, then the current phase is set to $\mathcal{B}$ and the pending node list is set to empty. This new state emulates the end of the bubble phase and will immediately transition to the default phase by rule `Bubble-to-Default`, thus skipping any remaining parts of the capture, target and bubble phases.

[3]In Safari, default actions are executed on the target node and its ancestors. The specification does not prescribe any specific list of nodes for the default phase.

`Run Event Handlers`: If there is no executing handler ($C_m = \phi$) and `stopImmediatePropagation` is unset, then this rule starts executing the next handler in the pending handler list $\mathcal{H}$. The notable aspect is that the $\mathcal{PC}$ of the new execution (the only frame in the new ρ) is obtained by joining the $\mathcal{PC}$s of the top frame of Σ ($!\Sigma.\ell$), the $\mathcal{PC}$ of the new handler ($\Gamma(f_m)$) and the $\mathcal{PC}$ when this handler was registered with this node (ℓ_c).

`Stop Immediate Propagation`: In the same initial state as the previous rule, if the `stopImmediatePropagation` flag of the event is set, then the current phase is set to $\mathcal{B}$ and the pending node list and the pending event handler list are both set to empty, directly bringing the state to the end of the bubble phase.

B. Correctness of IFC

We formally prove soundness of our IFC instrumentation of our model of the DOM, the event loop and the sequential JS semantics by proving the standard property called termination insensitive noninterference. This property says that two runs with equal low-observable inputs produce equal low-observable outputs. We assume that the IFC enforcement on the sequential JS semantics satisfies some basic properties, which are described in the appendix. Roughly, the assumptions say that every small step of the sequential JS semantics must preserve a standard bisimulation relation between two runs of the program starting from low-equal memories. We describe these assumptions explicitly in the appendix and later discharge them by for a specific instance of the sequential JS machine from [15] extended with DOM API. The details of that extension are provided in the appendix. Here, we focus on the proofs of noninterference for our reactive model of event loops. For simplicity, we consider only a two-point security lattice $L < H$, but our definitions and theorems generalize to arbitrary lattices.

As usual, we define equivalence $\kappa_1 \sim \kappa_2$ of states $\kappa = \langle \Sigma, \theta \rangle$ of our transition relation.[4] To do this, we must also define the equivalence of sequential machine configurations C, of node and event handler lists and of stack frames. We show these definitions below. The definition of heap equivalence $\theta_1 \sim \theta_2$ is inherited from the sequential JS model and says that the parts of the heaps θ_1 and θ_2 reachable from their global objects by traversing only L-labelled pointers must be equal. Similarly, we also inherit definitions of call stack equivalence and $\mathcal{PC}$ stack equivalence, $\sigma_1 \sim \sigma_2$ and $\rho_1 \sim \rho_2$, from the sequential JS model.

Definition 1. *Two machines C_1 and C_2 are equivalent, written $C_1 \sim C_2$, if either one of the following hold:*

[4]Technically, $\sim$ is parametrized by a partial bijection β between names of heap locations allocated at corresponding points in the two runs. However, we omit the partial bijection here for readability. The appendix resolves the notation.

1) $C_1 = \langle \iota_1, \sigma_1, \rho_1 \rangle$, $C_2 = \langle \iota_2, \sigma_2, \rho_2 \rangle$ *and*
   a) $(\rho_1 \sim \rho_2)$
   b) $(\sigma_1 \sim \sigma_2)$
   c) $(\Gamma(!\rho_1) = \Gamma(!\rho_2) = L \wedge \iota_1 = \iota_2)$ *or* $(\Gamma(!\rho_1) = \Gamma(!\rho_2) = H))$
2) $C_1 = C_2 = \phi$

Definition 2. *Two lists of nodes $\mathcal{N}_1$ and $\mathcal{N}_2$ are equivalent, written $\mathcal{N}_1 \sim \mathcal{N}_2$, iff $|\mathcal{N}_1| = |\mathcal{N}_2| \wedge \forall i \in |\mathcal{N}_1|.\ \mathcal{N}_1[i] \sim \mathcal{N}_2[i]$.*

Definition 3. *Two event handler lists $\mathcal{H}_1$ and $\mathcal{H}_2$ are equivalent, written $\mathcal{H}_1 \sim \mathcal{H}_2$, iff $|\mathsf{low}(\mathcal{H}_1)| = |\mathsf{low}(\mathcal{H}_2)|$ and for all i, $\mathsf{low}(\mathcal{H}_1)[i] \sim \mathsf{low}(\mathcal{H}_2)[i]$ where* low *is defined as:*

$$\mathsf{low}(nil) := nil$$

$$\mathsf{low}((\ell_c, eH) :: \mathcal{H}) := \begin{cases} eH :: \mathsf{low}(\mathcal{H}) & \text{if} \quad \ell_c = L \\ \mathsf{low}(\mathcal{H}) & \text{if} \quad \ell_c \neq L \end{cases}$$

The above definition states that two event handler lists are low-equal when the subsequences of low-visible event handlers in them are equal. (Note that event handlers eH are JS function objects, so the $\mathsf{low}(\mathcal{H}_1)[i] \sim \mathsf{low}(\mathcal{H}_2)[i]$ is the equivalence on JS objects from the sequential model.)

Next we define the equivalence of stack frames ν and the equivalence of stacks Σ. Intuitively, two frames are equivalent if they are labeled H or each of their respective elements are equivalent.

Definition 4. *Two frames ν_1 and ν_2 in the stack of machine configurations are equivalent, written $\nu_1 \sim \nu_2$, if either:*

1) $\Gamma(\nu_1) = H \wedge \Gamma(\nu_2) = H$ *or*
2) $\nu_1.C \sim \nu_2.C,\ \Gamma(\nu_1) = \Gamma(\nu_2),\ \nu_1.E \sim \nu_2.E,$ $\nu_1.N \sim \nu_2.N,\ \nu_1.\mathcal{A} \sim \nu_2.\mathcal{A},\ \nu_1.\mathcal{N} \sim \nu_2.\mathcal{N},$ $\nu_1.\mathcal{H} \sim \nu_2.\mathcal{H},\ \nu_1.\mathcal{P} = \nu_2.\mathcal{P}$ *and* *if* $\Gamma(!\rho(\nu_1.C)) = \Gamma(!\rho(\nu_2.C)) = L$, *then* $\nu_1.M = \nu_2.M$

Definition 5. *Given two stacks of machine configurations Σ_1 and Σ_2, suppose:*

1) *ν_1 is the first node in Σ_1 s.t. $\Gamma(\nu_1) = H$*
2) *ν_2 is the first node in Σ_2 s.t. $\Gamma(\nu_2) = H$*
3) *Σ_1' is the prefix of Σ_1 up to but not including ν_1*
4) *Σ_2' is the prefix of Σ_2 up to but not including ν_2*

Then, $\Sigma_1 \sim \Sigma_2$, iff (1) $|\Sigma_1'| = |\Sigma_2'|$, and (2) $\forall i \leq |\Sigma_1'|.\ (\Sigma_1'[i] \sim \Sigma_2'[i])$.

Definition 6 (κ-equivalence)**.** *Two states $\kappa = \langle \Sigma, \theta \rangle$ and $\kappa' = \langle \Sigma', \theta' \rangle$ are said to be equivalent, written $\kappa \sim \kappa'$, iff $\Sigma \sim \Sigma'$ and $\theta \sim \theta'$.*

We can now state our main noninterference theorem.

Theorem 1 (Termination-insensitive noninterference)**.** *If $\kappa_1 \sim \kappa_2$, $\kappa_1 \twoheadrightarrow_\alpha \kappa_1'$ and $\kappa_2 \twoheadrightarrow_\alpha \kappa_2'$, then either:*

- $\kappa_1' \sim \kappa_2$ *or*
- $\kappa_1 \sim \kappa_2'$.

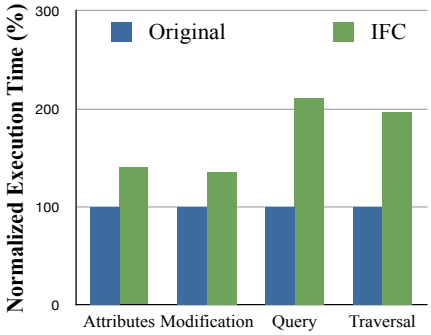

Figure 3: Overheads of IFC on Dromaeo DOM Core Benchmark Tests

We prove this theorem by setting a bisimulation relation $\kappa_1 \mathcal{R} \kappa_2$ which is equivalent to $\kappa_1 \sim \kappa_2$ and additionally provides enough structure to establish the theorem inductively. This relation is shown in the appendix, together with a proof of the theorem.

V. Implementation

We implement the IFC semantics described in Section IV in WebKit, a popular browser engine used in a number of browsers. Our implementation is built on top of the hybrid and optimized IFC implementation for JS bytecode from [15]. That implementation only instruments the sequential JS interpreter. We additionally instrument the DOM APIs, the event loop, and all native JS methods in the Array, RegExp, and String objects. Following [15], our implementation targets WebKit build #r122160 and works with the Safari web browser, version 6.0. All experiments are reported on a 3.2GHz Quad-core Intel Xeon processor with 8GB RAM, running Mac OS X version 10.7.4. Since [15] and our extension only target the bytecode interpreter, we disable JIT in all our experiments.

We attach security labels to every node in the DOM graph and all its properties, including pointer to other nodes. We then add appropriate IFC checks in the native C code implementing all DOM APIs up to Level 3. Additional instrumentation carries the labels from the native C code to the JS interpreter, where IFC checks are already provided by [15]. We make similar changes to the event loop, labeling every event and event handler. Our work adds approximately 2,300 lines of code above the 4,500 lines of initial instrumentation from [15]. Our implementation stops the execution of the program when it detects an IFC violation but for the purpose of measuring performance overheads reported here, we ignore violations.

We evaluate our instrumentation on the Dromaeo DOM Core benchmark [44], which measures the performance

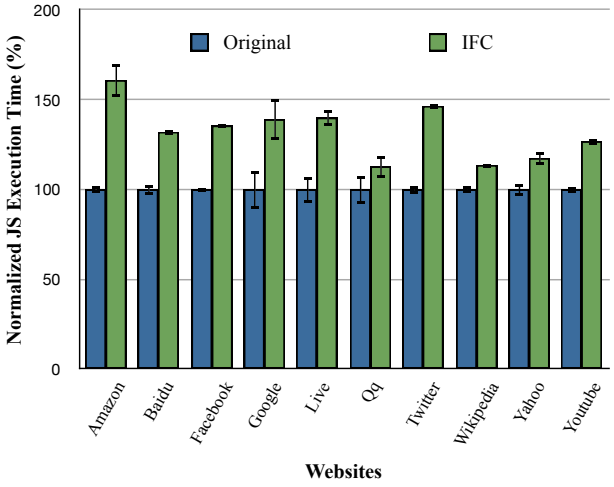

Figure 4: Overheads of IFC on Alexa Top 10 Websites

of various operations on the DOM. We find an average overhead of approximately 71% over the uninstrumented browser. Normalized overheads on different kinds of tests are shown in Figure 3 (standard deviations on individual tests were small, ranging from 0.17% to 8.45%). To get a more realistic evaluation, we also tested our instrumentation on the Alexa Top 10 websites [45]. We measured the execution time of the JS that loads initially on each website's front page, without any user interaction. The graph in Figure 4 shows normalized execution time. Error bars are standard deviations. The average overhead is approximately 32% and the worst overhead is around 60%. Note that both the Dromaeo and Alexa tests are very performance-intensive and do not count common browser delays like network communication and page rendering in the baseline. Compared to a baseline that includes these delays, our overheads are negligible.

Finally, as a sanity check, we run the very popular SunSpider benchmark [46]. SunSpider is a pure JS benchmark that does not cover events or the DOM, so it does not really execute the code paths we have instrumented. As a result, the overheads we get are similar to those of [15]. These overheads are shown in the appendix.

VI. Related Work

We compare briefly to the most closely related work. Russo *et al.* [23] developed the first dynamic IFC monitor for an imperative language with DOM-like trees. In particular, they highlight information leaks via deletion and navigation of nodes in the DOM and show how their monitor can prevent them. The work does not cover live collections or event handling. Almeida-Matos *et al.* [24] provide an IFC monitor for a subset of the DOM (for a language similar to [23]) and also extend it for live collections. Their work covers only a part of the DOM and requires programmer provided annotations for handling live collections, which our method does not, as explained in Section III(c).

Hedin *et al.* [14], [20] develop a source-level interpreter for a core of JS with dynamic IFC checks. The interpreter is written from-scratch and provides an alternate code path for JS execution in a browser through a plug-in. In comparison, we build on [15], which works at the level of JS bytecode in WebKit and uses native WebKit code paths (which adds several order of magnitude less overhead). Besides, that work is limited to core JS without the DOM or events. COWL [19] is a recent system for improving security by confining JavaScript in web browsers. COWL uses coarse-grained labels at the level of browsing context (page, frame or workers) unlike our design that relies on fine-grained labels and, hence, offers higher precision. Kerschbaumer et al. [16] build an implementation of an information flow monitor for WebKit but do not handle all implicit flows. A black-box approach to enforcing non-interference is based on secure multi-execution (SME) [36]. Bielova *et al.* [17] and De Groef *et al.* [18] implement SME for web browsers. These systems do not attach labels to specific fields in the DOM. Instead, labels are attached to individual DOM APIs.

Gardner et al. [25] developed a formal specification for a minimal subset of DOM Level 1. Our formal specification goes beyond this and covers up to DOM Level 3. Lerner et al. [47] develop a formal model for the event handling mechanism of web browsers, but do not consider IFC. Our model additionally adds support for preemption of event handlers. The idea of developing a formal model for web browsers traces lineage to Featherweight Firefox [26], an OCaml model of the reactive core of a web browser. Both the DOM and the event handling mechanism are modeled, but abstractly, e.g., preemption is not modeled, nor are many DOM APIs.

VII. Conclusion

Although information flow control (IFC) for web browsers is a well-studied topic, two central browser aspects — event handling and the DOM — have not received enough attention. As we observe, both event handling and the DOM can be a source of subtle information flow leaks. Accordingly, we develop a formal model of event handling with preemption and of the DOM APIs up to Level 3, both fully instrumented for IFC. We prove our instrumentation sound, implement our ideas in WebKit and observe moderate overhead due to our IFC checks.

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within the Centre for IT-Security, Privacy and Accountability (CISPA) at Saarland University.

REFERENCES

[1] D. Jang, R. Jhala, S. Lerner, and H. Shacham, "An empirical study of privacy-violating information flows in JavaScript web applications," in *Proc. ACM Conference on Computer and Communications Security*, 2010, pp. 270–283.

[2] J. R. Mayer and J. C. Mitchell, "Third-party web tracking: Policy and technology," in *Proc. IEEE Symposium on Security and Privacy*, 2012, pp. 413–427.

[3] C. Yue and H. Wang, "Characterizing insecure JavaScript practices on the Web," in *Proc. International World Wide Web Conference*, 2009, pp. 961–970.

[4] N. Nikiforakis, L. Invernizzi, A. Kapravelos, S. Van Acker, W. Joosen, C. Kruegel, F. Piessens, and G. Vigna, "You are what you include: Large-scale evaluation of remote JavaScript inclusions," in *Proc. ACM Conference on Computer and Communications Security*, 2012, pp. 736–747.

[5] A. Barth, "The web origin concept." [Online]. Available: http://tools.ietf.org/html/rfc6454

[6] "Content Security Policy 1.0." [Online]. Available: http://www.w3.org/TR/CSP/

[7] G. Richards, C. Hammer, B. Burg, and J. Vitek, "The eval that men do – a large-scale study of the use of eval in JavaScript applications," in *Proc. European Conference on Object-Oriented Programming*, 2011, pp. 52–78.

[8] "Google Caja - A source-to-source translator for securing JavaScript-based web content." [Online]. Available: http://code.google.com/p/google-caja/

[9] "Facebook. FBJS." [Online]. Available: https://developers.facebook.com/docs/javascript

[10] D. Crockford, "ADSafe." [Online]. Available: http://adsafe.org/

[11] S. Maffeis and A. Taly, "Language-based isolation of untrusted javascript," in *Proc. IEEE Computer Security Foundations Symposium*, 2009, pp. 77–91.

[12] P. Agten, S. Van Acker, Y. Brondsema, P. H. Phung, L. Desmet, and F. Piessens, "JSand: Complete client-side sandboxing of third-party JavaScript without browser modifications," in *Proc. Annual Computer Security Applications Conference*, 2012, pp. 1–10.

[13] S. Just, A. Cleary, B. Shirley, and C. Hammer, "Information flow analysis for JavaScript," in *Proc. ACM SIGPLAN International Workshop on Programming Language and Systems Technologies for Internet Clients*, 2011, pp. 9–18.

[14] D. Hedin and A. Sabelfeld, "Information-flow security for a core of JavaScript," in *Proc. IEEE Computer Security Foundations Symposium*, 2012, pp. 3–18.

[15] A. Bichhawat, V. Rajani, D. Garg, and C. Hammer, "Information flow control in WebKit's JavaScript bytecode," in *Proc. Principles of Security and Trust*, 2014, pp. 159–178.

[16] C. Kerschbaumer, E. Hennigan, P. Larsen, S. Brunthaler, and M. Franz, "Towards precise and efficient information flow control in web browsers," in *Proc. Trust and Trustworthy Computing*, 2013, pp. 187–195.

[17] N. Bielova, D. Devriese, F. Massacci, and F. Piessens, "Reactive non-interference for a browser model," in *Proc. International Conference on Network and System Security*, 2011, pp. 97–104.

[18] W. De Groef, D. Devriese, N. Nikiforakis, and F. Piessens, "Flowfox: a web browser with flexible and precise information flow control," in *Proc. ACM Conference on Computer and Communications Security*, 2012, pp. 748–759.

[19] D. Stefan, E. Z. Yang, P. Marchenko, A. Russo, D. Herman, B. Karp, and D. Mazières, "Protecting users by confining JavaScript with COWL," in *Proc. USENIX Symposium on Operating Systems Design and Implementation*, 2014, pp. 131–146.

[20] D. Hedin, A. Birgisson, L. Bello, and A. Sabelfeld, "JSFlow: Tracking information flow in JavaScript and its APIs," in *Proc. ACM Symposium on Applied Computing*, 2014, pp. 1663–1671.

[21] "Html 5 spec." [Online]. Available: http://www.w3.org/TR/html5

[22] "W3C Document Object Model Level 3 Core Specification." [Online]. Available: http://www.w3.org/TR/DOM-Level-3-Core

[23] A. Russo, A. Sabelfeld, and A. Chudnov, "Tracking information flow in dynamic tree structures," in *Proc. European Symposium on Research in Computer Security*, 2009, pp. 86–103.

[24] A. Almeida-Matos, J. Fragoso Santos, and T. Rezk, "An information flow monitor for a core of dom," in *Proc. Trustworthy Global Computing*, 2014, pp. 1–16.

[25] P. Gardner, G. Smith, M. J. Wheelhouse, and U. Zarfaty, "DOM: Towards a formal specification," in *Proc. ACM SIGPLAN Workshop on Programming Language Technologies for XML*, 2008.

[26] A. Bohannon and B. C. Pierce, "Featherweight Firefox: Formalizing the core of a web browser," in *Proc. USENIX conference on Web Application Development*, 2010, pp. 11–22.

[27] A. Askarov, S. Hunt, A. Sabelfeld, and D. Sands, "Termination-insensitive noninterference leaks more than just a bit," in *Proc. European Symposium on Research in Computer Security*, 2008, pp. 333–348.

[28] D. E. Denning and P. J. Denning, "Certification of programs for secure information flow," *Communications of the ACM*, vol. 20, no. 7, pp. 504–513, Jul. 1977.

[29] D. Volpano, C. Irvine, and G. Smith, "A sound type system for secure flow analysis," *Journal of Computer Security*, vol. 4, no. 2-3, pp. 167–187, Jan 1996.

[30] S. Hunt and D. Sands, "On flow-sensitive security types," in *Proc. ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, 2006, pp. 79–90.

[31] A. C. Myers, "JFlow: Practical mostly-static information flow control," in *Proc. ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, 1999, pp. 228–241.

[32] T. H. Austin and C. Flanagan, "Efficient purely-dynamic information flow analysis," in *Proc. ACM SIGPLAN Workshop on Programming Languages and Analysis for Security*, 2009, pp. 113–124.

[33] ——, "Permissive dynamic information flow analysis," in *Proc. ACM SIGPLAN Workshop on Programming Languages and Analysis for Security*, 2010, pp. 1–12.

[34] A. Askarov and A. Sabelfeld, "Tight enforcement of information-release policies for dynamic languages," in *Proc. IEEE Computer Security Foundations Symposium*, 2009, pp. 43–59.

[35] A. Sabelfeld and A. Russo, "From dynamic to static and back: Riding the roller coaster of information-flow control research," in *Proc. Perspectives of Systems Informatics*, 2010, pp. 352–365.

[36] D. Devriese and F. Piessens, "Noninterference through secure multi-execution," in *Proc. IEEE Symposium on Security and Privacy*, 2010, pp. 109–124.

[37] F. Nentwich, N. Jovanovic, E. Kirda, C. Kruegel, and G. Vigna, "Cross-site scripting prevention with dynamic data tainting and static analysis," in *Proc. Network and Distributed System Security Symposium*, 2007.

[38] G. L. Guernic, A. Banerjee, T. Jensen, and D. A. Schmidt, "Automata-based confidentiality monitoring," in *Proc. Asian Computing Science Conference on Secure Software*, 2006, pp. 75–89.

[39] G. Le Guernic, "Automaton-based confidentiality monitoring of concurrent programs," in *Proc. IEEE Computer Security Foundations Symposium*, 2007, pp. 218–232.

[40] J. A. Goguen and J. Meseguer, "Security policies and security models," in *Proc. IEEE Symposium on Security and Privacy*, 1982, pp. 11–20.

[41] A. Russo and A. Sabelfeld, "Dynamic vs. static flow-sensitive security analysis," in *Proc. IEEE Computer Security Foundations Symposium*, 2010, pp. 186 –199.

[42] S. A. Zdancewic, "Programming languages for information security," Ph.D. dissertation, Cornell University, August 2002.

[43] "W3C Document Object Model Level 3 Events Specification." [Online]. Available: http://www.w3.org/TR/DOM-Level-3-Events

[44] "Dromaeo: JS performance testing." [Online]. Available: http://dromaeo.com/

[45] "Alexa top 500 global sites." [Online]. Available: http://www.alexa.com/topsites

[46] "Sunspider 1.0.2 javascript benchmark." [Online]. Available: http://www.webkit.org/perf/sunspider/sunspider.html

[47] B. S. Lerner, M. J. Carroll, D. P. Kimmel, H. Q. de la Vallee, and S. Krishnamurthi, "Modeling and reasoning about dom events," in *Proc. USENIX Conference on Web Application Development*, 2012, pp. 1–12.

VIII. Appendix

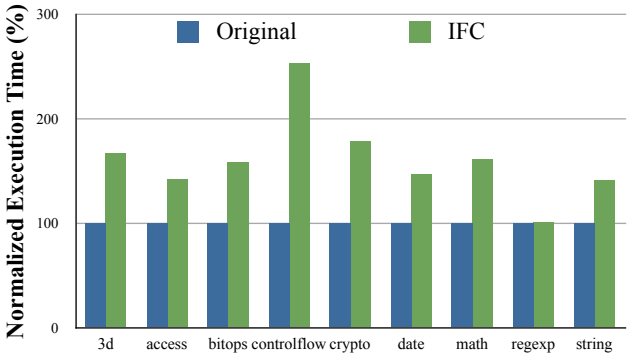

Figure 5: Overheads of IFC on SunSpider JavaScript Benchmark Tests

A. Proofs of IFC for Event handling

Definition 1. *Two machines C_1 and C_2 are low-equivalent, written $C_1 \sim^\beta C_2$, if either:*

1) $C_1 = \langle \iota_1, \sigma_1, \rho_1 \rangle$, $C_2 = \langle \iota_2, \sigma_2, \rho_2 \rangle$, *and* $(\rho_1 \sim^\beta \rho_2) \wedge (\sigma_1 \sim^\beta \sigma_2) \wedge (\Gamma(!\rho_1) = \Gamma(!\rho_2) = L \implies \iota_1 = \iota_2)$ *or*
2) $C_1 = C_2 = \phi$

Definition 2. *Two lists of nodes $\mathcal{N}_1$ and $\mathcal{N}_2$ are equivalent, written $\mathcal{N}_1 \sim^\beta \mathcal{N}_2$, iff $|\mathcal{N}_1| = |\mathcal{N}_2| \wedge \forall i \in |\mathcal{N}_1|.\ \mathcal{N}_1[i] \sim^\beta \mathcal{N}_2[i]$.*

Definition 3. *Two event handler lists $\mathcal{H}_1$ and $\mathcal{H}_2$ are equivalent, written $\mathcal{H}_1 \sim^\beta \mathcal{H}_2$, iff $|\mathsf{low}(\mathcal{H}_1)| = |\mathsf{low}(\mathcal{H}_2)|$ and for all i, $\mathsf{low}(\mathcal{H}_1)[i] \sim^\beta \mathsf{low}(\mathcal{H}_2)[i]$ where low is defined as:*

$$\mathsf{low}(nil) := nil$$

$$\mathsf{low}((\ell_c, eH) :: \mathcal{H}) := \begin{cases} eH :: \mathsf{low}(\mathcal{H}) & \text{if} \quad \ell_c = L \\ \mathsf{low}(\mathcal{H}) & \text{if} \quad \ell_c \neq L \end{cases}$$

Definition 4. *Two frames ν_1 and ν_2 in the stack of machine configurations are equivalent, written $\nu_1 \sim^\beta \nu_2$, if either:*

1) $\Gamma(\nu_1) = H \wedge \Gamma(\nu_2) = H$ *or*
2) $\nu_1.C \sim^\beta \nu_2.C,\ \Gamma(\nu_1) = \Gamma(\nu_2),\ \nu_1.E \sim^\beta \nu_2.E,$ $\nu_1.N \sim^\beta \nu_2.N,\ \nu_1.\mathcal{A} \sim^\beta \nu_2.\mathcal{A},\ \nu_1.\mathcal{N} \sim^\beta \nu_2.\mathcal{N},$ $\nu_1.\mathcal{H} \sim^\beta \nu_2.\mathcal{H},\ \nu_1.\mathcal{P} = \nu_2.\mathcal{P}$ *and*

if $\Gamma(!\rho(\nu_1.C)) = \Gamma(!\rho(\nu_2.C)) = L$, *then* $\nu_1.M = \nu_2.M$

Definition 5. *Given two stacks of machine configurations* Σ_1 *and* Σ_2, *suppose:*

1) ν_1 *is the first node in* Σ_1 *s.t.* $\Gamma(\nu_1) = H$
2) ν_2 *is the first node in* Σ_2 *s.t.* $\Gamma(\nu_2) = H$
3) Σ_1' *is the prefix of* Σ_1 *up to but not including* ν_1
4) Σ_2' *is the prefix of* Σ_2 *up to but not including* ν_2

Then, $\Sigma_1 \sim^\beta \Sigma_2$, *iff (1)* $|\Sigma_1'| = |\Sigma_2'|$, *and (2)* $\forall i \leq |\Sigma_1'|.\ (\Sigma_1'[i] \sim^\beta \Sigma_2'[i])$.

Definition 6 (κ-equivalence)**.** *Two states* $\kappa = \langle \Sigma, \theta \rangle$ *and* $\kappa' = \langle \Sigma', \theta' \rangle$ *are said to be equivalent, written* $\kappa \sim^\beta \kappa'$, *iff* $\Sigma \sim^\beta \Sigma'$ *and* $\theta \sim^\beta \theta'$.

Definition 7. *The function* $\Gamma(\kappa)$ *for a state* $\kappa = \langle \Sigma, \theta \rangle$ *is defined as:*
$\Gamma(\kappa) = \Gamma(!\Sigma) \sqcup \ell_c$ *where* $\ell_c = (\textit{if } (!\Sigma.C \neq \phi) \textit{ then } \Gamma(!\rho(!\Sigma.C)) \textit{ else } \bot)$

Definition 8 (Transitions)**.** *Transitions* $\twoheadrightarrow^H_\alpha$ *and* $\twoheadrightarrow^L_\alpha$ *are defined as:*

1) $\kappa_1 \twoheadrightarrow^H_\alpha \kappa_2$: $\Gamma(\kappa_1) \geq H$
2) $\kappa_1 \twoheadrightarrow^L_\alpha \kappa_2$: $\Gamma(\kappa_1) = L$

Definition 9 (Bisimulation)**.** *A relation* $\mathcal{R}$ *between configurations* $(\kappa := \langle \Sigma, \theta \rangle)$ *is called a bisimulation, written* $\kappa_1 \mathcal{R} \kappa_2$, *iff:*

1) $\kappa_1 \sim^\beta \kappa_2$
2) $\kappa_1 \twoheadrightarrow^H_\alpha \kappa_1' \implies \kappa_1' \mathcal{R} \kappa_2$
3) $\kappa_2 \twoheadrightarrow^H_\alpha \kappa_2' \implies \kappa_1 \mathcal{R} \kappa_2'$
4) $\kappa_1 \twoheadrightarrow^L_\alpha \kappa_1' \implies \Gamma(\kappa_1') \geq H \implies \kappa_1' \mathcal{R} \kappa_2$
5) $\kappa_2 \twoheadrightarrow^L_\alpha \kappa_2' \implies \Gamma(\kappa_2') \geq H \implies \kappa_1 \mathcal{R} \kappa_2'$
6) $\kappa_1 \twoheadrightarrow^L_\alpha \kappa_1' \implies \kappa_2 \twoheadrightarrow^L_\alpha \kappa_2' \implies \Gamma(\kappa_1') = \Gamma(\kappa_2') \implies \kappa_1' \mathcal{R} \kappa_2'$

Assumption 1 (Sequential Machine - Confinement Lemma)**.** *Suppose* $C = \langle \iota, \sigma, \rho \rangle$, $C' = \langle \iota', \sigma', \rho' \rangle$, $\langle \theta, C \rangle \rightarrow \langle \theta', C' \rangle$ *and* $\Gamma(!\rho) = H$, *then* $\rho \sim \rho'$, $\sigma \sim^\beta_{\rho,\rho'} \sigma'$ *and* $\theta \sim^\beta \theta'$

Assumption 2 (Sequential Machine - Supporting Lemma 1)**.**
Suppose $C_1 = \langle \iota, \sigma_1, \rho_1 \rangle$, $C_2 = \langle \iota, \sigma_2, \rho_2 \rangle$,
$C_1' = \langle \iota_1', \sigma_1', \rho_1' \rangle$, $C_2' = \langle \iota_2', \sigma_2', \rho_2' \rangle$
$\langle \theta_1, C_1 \rangle \rightarrow \langle \theta_1', C_1' \rangle$,
$\langle \theta_2, C_2 \rangle \rightarrow \langle \theta_2', C_2' \rangle$,
$\rho_1 \sim \rho_2$, $\Gamma(!\rho_1) = \Gamma(!\rho_2) = L$, $\Gamma(!\rho_1') = \Gamma(!\rho_2')$ *and* $(\sigma_1 \sim^\beta_{\rho_1,\rho_2} \sigma_2) \wedge (\theta_1 \sim^\beta \theta_2)$
then $\rho_1' \sim \rho_2'$, *and* $(\sigma_1' \sim^\beta_{\rho_1',\rho_2'} \sigma_2') \wedge (\theta_1' \sim^\beta \theta_2'))$.

Assumption 3 (Sequential Machine - Supporting Lemma 2)**.**
Suppose
$C_0' = \langle \iota_0, \sigma_0', \rho_0' \rangle$, $C_0'' = \langle \iota_0, \sigma_0'', \rho_0'' \rangle$,
$C_1' = \langle \iota_1', \sigma_1', \rho_1' \rangle$, $C_2' = \langle \iota_2', \sigma_2', \rho_2' \rangle$,
$C_n' = \langle \iota_n', \sigma_n', \rho_n' \rangle$, $C_m'' = \langle \iota_m'', \sigma_m'', \rho_m'' \rangle$ *and*

1) $\langle \theta_0', C_0' \rangle \rightarrow \langle \theta_1', C_1' \rangle \rightarrow^{n-1} \langle \theta_n', C_n' \rangle$,
2) $\langle \theta_0'', C_0'' \rangle \rightarrow \langle \theta_1'', C_1'' \rangle \rightarrow^{m-1} \langle \theta_m'', C_m'' \rangle$,
3) $(\rho_0' \sim \rho_0'')$, $(\sigma_0' \sim^\beta_{\rho_0',\rho_0''} \sigma_0'')$, $(\theta_0' \sim^\beta \theta_0'')$,
4) $(\Gamma(!\rho_0') = \Gamma(!\rho_0'') = L)$, $(\Gamma(!\rho_n') = \Gamma(!\rho_m'') = L)$,
5) $\forall (0 < i < n).(\Gamma(!\rho_i') = H) \wedge \forall (0 < j < m).(\Gamma(!\rho_j'') = H)$,

then
$(\iota_n' = \iota_m'')$, $(\rho_n' \sim \rho_m'')$, $(\sigma_n' \sim^\beta_{\rho_n',\rho_m''} \sigma_m'')$, *and* $(\theta_n' \sim^\beta \theta_m'')$.

We discharge these assumptions later in the appendix (by proving the lemma 4, lemma 5 and lemma 6) for a specific instance of the sequential machine from [15] extended with the proofs for the DOM API.

Lemma 1 (Confinement)**.** *Given* $(\kappa_1 = \langle \Sigma_1, \theta_1 \rangle) \sim^\beta (\kappa_2 = \langle \Sigma_2, \theta_2 \rangle)$ *and* $\kappa_1 \twoheadrightarrow^H_\alpha \kappa_1'$ *then* $\kappa_1' \sim^\beta \kappa_2$

Proof: Say $\kappa_1' = \Sigma_1', \theta_1'$ To prove: $\Sigma_1' \sim^\beta \Sigma_2$ and $\theta_1' \sim^\beta \theta_2$.
Proof by case analysis on the derivation rules of the concurrent machine:

1) Local Computation-no dispatch: $\theta_1 \sim^\beta \theta_1'$ from confinement lemma (Lemma 4) of sequential machine and $\theta_1 \sim^\beta \theta_2$, so $\theta_1' \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   - If $\Gamma(!\Sigma_1) \geq H$, then the low part of Σ_1 remains the same. Hence, $\Sigma_1' \sim^\beta \Sigma_2$
   - If $\Gamma(!\Sigma_1) = L \wedge !\Sigma_1.C \neq \phi \wedge \Gamma(!\rho(!\Sigma_1.C)) \geq H$, then $!\Sigma_2.C \neq \phi \wedge \Gamma(!\rho(!\Sigma_2.C)) \geq H$. From confinement and supporting lemma 2 (Lemmas 4 and 6), $!\Sigma_1'.C \sim^\beta !\Sigma_2.C$. Other parts of Σ_1 remain unchanged in Σ_1'. Thus, $\Sigma_1' \sim^\beta \Sigma_2$.
2) Local Computation-dispatch:
   $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   a) If $\Gamma(!\Sigma_1) \geq H$, then $\Gamma(!\Sigma_1') \geq H$. Thus, the low parts of Σ_1 remain unchanged. Hence, $\Sigma_1' \sim^\beta \Sigma_2$.
   b) If $\Gamma(!\Sigma_1) = L \wedge !\Sigma_1.C \neq \phi \wedge \Gamma(!\rho(!\Sigma_1.C)) \geq H$, then $\Gamma(!\rho(!\Sigma_2.C)) \geq H$. From Lemmas 4 and 6, $!\Sigma_1''.C \sim^\beta !\Sigma_2.C$ and the remaining low part of Σ_1 remains unchanged. By definition 1, 4 and 5, $\Sigma_1' \sim^\beta \Sigma_2$.
3) Preemption-point: $(\theta_1' = \theta_1) \sim^\beta \theta_2$. To show: $\Sigma_1' \sim^\beta \Sigma_2$
   a) If $\Gamma(!\Sigma_1) \geq H$, then $\Gamma(!\Sigma_1') \geq H$. Thus, the low parts of Σ_1 remain unchanged. Hence, $\Sigma_1' \sim^\beta \Sigma_2$.
   b) If $\Gamma(!\Sigma_1) = L \wedge C_1 \neq \phi \wedge \Gamma(!C_1.\rho) \geq H$, then $\Gamma(!C_2.\rho) \geq H$. From Lemmas 4 and 6, $!\Sigma_1'.C \sim^\beta !\Sigma_2.C$ and the remaining low part of Σ_1 remains unchanged. By definition 1, 4 and 5, $\Sigma_1' \sim^\beta \Sigma_2$.
4) End: $\theta_1' = \theta_1 \sim^\beta \theta$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   a) If $\Gamma(!\Sigma_1) \geq H$ then $\Gamma(!\Sigma_1') \geq H$. Thus, $\Sigma_1' \sim^\beta \Sigma_2$.
   b) If $\Gamma(!\Sigma_1) = L$. As *end* is the last instruction, $\rho(!\Sigma_1.C_1)$ is empty from sequential machine semantics.
5) Fire Event: $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   a) If $\Gamma(!\Sigma_1) \geq H$, then $\Gamma(!\Sigma_1') \geq H$. Thus, the low parts of Σ_1 remain unchanged. Hence, $\Sigma_1' \sim^\beta \Sigma_2$.

b) If $!\Sigma_1.C \neq \phi \wedge \Gamma(!\rho(!\Sigma_1.C)) \geq H$, then $\Gamma(!\Sigma_1') \geq H$. The low part of Σ_1 remains the same, i.e., Σ_1 is unchanged in Σ_1'. As $\Sigma_1 \sim^\beta \Sigma_2$, $\Sigma_1' \sim^\beta \Sigma_2$.

c) $\Gamma(!\Sigma_1) = L \wedge \Gamma(\alpha) \geq H$: $\Gamma(!\Sigma_1') \geq H$. The low part of Σ_1 remains the same. As $\Sigma_1 \sim^\beta \Sigma_2$, $\Sigma_1' \sim^\beta \Sigma_2$.

6) Start-to-Capture, Capture-to-Target, Target-to-Bubble, Bubble-to-Default, Default Prevented, Get Event Handlers, Stop Propagation, Run Event Handlers, Stop Immediate Propagation: $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   a) If $\Gamma(!\Sigma_1) \geq H$, then $\Gamma(!\Sigma_1') \geq H$. Thus, the low parts of Σ_1 remain unchanged. Hence, $\Sigma_1' \sim^\beta \Sigma_2$.
   b) $!\Sigma_1.C = \phi$ so it can't be the case that $\Gamma(!\Sigma_1) = L$
7) Remove Frame: $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   a) If $\Gamma(!\Sigma_1) \geq H$, then $\Gamma(!\Sigma_1') = L$ or $\Gamma(!\Sigma_1') = H$. If $\Gamma(!\Sigma_1') = L$, then it is the last L labeled frame in Σ_1. Thus, the low parts of Σ_1 remain unchanged. Else if $\Gamma(!\Sigma_1') = H$, still the low parts of Σ_1 remain unchanged. Hence, $\Sigma_1' \sim^\beta \Sigma_2$.
   b) $!\Sigma_1.C = \phi$
8) Run Suspended Machine: $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$:
   a) If $\Gamma(!\Sigma_1) \geq H$, then $\Gamma(!\Sigma_1') \geq H$. Thus, the low parts of Σ_1 remain unchanged. Hence, $\Sigma_1' \sim^\beta \Sigma_2$.
   b) $\Gamma(!\rho(!\Sigma_1.C)) \geq H$: Σ_1' remains unchanged from Σ_1 except for $!\Sigma_1.M$. By definition 4, $!\Sigma_1' \sim^\beta !\Sigma_2$. Thus, $\Sigma_1' \sim^\beta \Sigma_2$.

■

Lemma 2. *Let* $(\kappa_1 = \langle \Sigma_1, \theta_1 \rangle) \sim^\beta (\kappa_2 = \langle \Sigma_2, \theta_2 \rangle)$. *If* $\kappa_1 \twoheadrightarrow_\alpha^L \kappa_1'$ *and* $\Gamma(\kappa_1') \geq H$ *then* $\kappa_1' \sim^\beta \kappa_2$

Proof: Let $\kappa_1' = \Sigma_1', \theta_1'$ To prove: $\Sigma_1' \sim^\beta \Sigma_2$ and $\theta_1' \sim^\beta \theta_2$.

Proof by case analysis on the derivation rules of the concurrent machine:

1) Local Computation-no dispatch, Preemption-point: Apply on branch rules as $\Gamma(\kappa_1') \geq H$. Thus, $\theta_1 = \theta_1'$ and $\sigma(!\Sigma_1.C) = \sigma(!\Sigma_1'.C)$. $\theta_1' = \theta_1 \sim^\beta \theta_2$. $\sigma(!\Sigma_1'.C) = \sigma(!\Sigma_1.C) \sim^\beta \sigma(!\Sigma_2.C)$. Thus, $\Sigma_1' \sim^\beta \Sigma_2$.
2) Local Computation-dispatch, Fire Event: $!\Sigma_1'.C = \phi$ hence $\Gamma(!\Sigma_1') \geq H$.
3) End, Start-to-Capture, Capture-to-Target, Target-to-Bubble, Bubble-to-Default, Default Prevented, Remove Frame, Get Event Handlers, Stop Propagation, Stop Immediate Propagation: Since $!\Sigma_1'.C = \phi$ therefore $\Gamma(!\Sigma_1') \geq H$ and that can happen only if $\Gamma(!\Sigma_1) \geq H$. Hence case doesn't apply.
4) Run Event Handlers: $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$. Since, $\Gamma(!\Sigma_1) = L$ therefore $\Gamma(f_m) \geq H$ and hence $\Gamma(!\rho(!\Sigma_1'.C)) \geq H$. As the first node of $\rho(!\Sigma_1'.C)$ is labeled H, we do not compare the call-stacks (σ). Also $!\Sigma_1'.\mathcal{H} \sim^\beta !\Sigma_2.\mathcal{H}$. Thus, $\Sigma_1' \sim^\beta \Sigma_2$.
5) Run Suspended Machine: $\theta_1' = \theta_1 \sim^\beta \theta_2$. To show $\Sigma_1' \sim^\beta \Sigma_2$: $\rho(!\Sigma_1'.C) = \rho(!\Sigma_1.C) \sim^\beta \rho(!\Sigma_2.C)$. Thus, $\Gamma(\kappa_1) \geq H$. (Case does not apply)

■

Lemma 3 (Simulation)**.** *Given* $(\kappa_1 = \langle \Sigma_1, \theta_1 \rangle) \sim^\beta (\kappa_2 = \langle \Sigma_2, \theta_2 \rangle)$.
If $\kappa_1 \twoheadrightarrow_\alpha^L \kappa_1'$, $\kappa_2 \twoheadrightarrow_\alpha^L \kappa_2'$ *and* $\Gamma(\kappa_1') = \Gamma(\kappa_2')$ *then* $\exists \beta' : ((\beta' \supseteq \beta) \wedge \kappa_1' \sim^{\beta'} \kappa_2'$

Proof: Let $\kappa_1' = \langle \Sigma_1', \theta_1' \rangle$ and $\kappa_2' = \langle \Sigma_2', \theta_2' \rangle$.
To prove: $\Sigma_1' \sim^\beta \Sigma_2'$ and $\theta_1' \sim^\beta \theta_2'$.
Case analysis on the derivation rules of the concurrent machine:

1) Local Computation-no dispatch: For any new objects a and b that are created, $\beta' = \beta \cup (a, b)$. From supporting lemma 1 (Lemma 5) $\theta_1' \sim^{\beta'} \theta_2'$ and $!\Sigma_1'.C \sim^{\beta'} !\Sigma_2'.C$. Thus, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
2) Local Computation-dispatch: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. Σ_1 and Σ_2 are updated in 2 steps:
   - $!\Sigma_1''.C \sim^{\beta'} !\Sigma_2''.C$ from Lemma 5 and $!\Sigma_1''.M = !\Sigma_2''.M = S$. Thus, $\Sigma_1'' \sim^{\beta'} \Sigma_2''$.
   - Let the new frames pushed on top of Σ_1'' and Σ_2'' be ν_1 and ν_2, respectively. $\nu_1.C = \nu_2.C = \phi$, $\nu_1.E = (\sigma(!\Sigma_1.C)).arg[1] \sim^{\beta'} (\sigma(!\Sigma_1.C)).arg[1] = \nu_2.E$, $\nu_1.N = (\sigma(!\Sigma_1.C)).arg[0] \sim^{\beta'} (\sigma(!\Sigma_1.C)).arg[0] = \nu_2.N$, $\nu_1(aL) = getAncestors(\nu_1.N, \theta_1) \sim^{\beta'} getAncestors(\nu_2.N, \theta_2) = \nu_2(aL)$, $\nu_1(\mathcal{N}) = \nu_2(\mathcal{N}) = []$, $\nu_1(\mathcal{H}) = \nu_2(\mathcal{H}) = []$ and $\nu_1(P) = \nu_2(P) = \mathcal{S}$. For $\nu_1.\ell$ and $\nu_2.\ell$ the following cases arise from the heap-equivalence definition:
     – The path from o to root consists of only L pointers (pointer label corresponds to the $\Gamma(parentLabel)$) in both cases. In that case $\nu_1.\ell = \nu_2.\ell = L$.
     – There is at least one H pointer in the path from o to the root. In that case $\nu_1.\ell \geq H$ and $\nu_2.\ell \geq H$.

   Hence, $\nu_1.\ell \sim^{\beta'} \nu_2.\ell$. Therefore, $\nu_1 \sim^{\beta'} \nu_2$ from Definition 4.

   Thus, $\Sigma_1' \sim^{\beta'} \Sigma_2'$ by Definition 5.
3) Preemption-point: For any new objects a and b that are created, $\beta' = \beta \cup (a, b)$. Thus, $\theta_1' \sim^{\beta'} \theta_2'$ by Lemma 5. $\Sigma_1' := \Sigma_1[!\Sigma_1.C := C_1', !\Sigma_1.M := S]$ and $\Sigma_2' := \Sigma[!\Sigma_2.C := C_2', !\Sigma_1.M := S]$. Since $C_1' \sim^{\beta'} C_2'$ (Lemma 5), $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
4) End: $\beta' = \beta$. $\theta_1' \sim^{\beta'} \theta_2'$ by Lemma 5. $\Sigma_1' := \Sigma_1[!\Sigma_1.C := \phi, !\Sigma_1.M := S]$ and $\Sigma_2' := \Sigma[!\Sigma_2.C := \phi, !\Sigma_1.M := S]$. Thus, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
5) Fire Event: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. The event and node objects, e_1, e_2 and o_1, o_2 are under partial bijection β, i.e., $(e_1, e_2) \in \beta$ and $(o_1, o_2) \in \beta$. Let

the new frames pushed on top of Σ_1 and Σ_2 be ν_1 and ν_2, respectively. $\nu_1.C = \nu_2.C = \phi$, $\nu_1.E = (\sigma(!\Sigma_1.C)).arg[1] \sim^{\beta'} (\sigma(!\Sigma_1.C)).arg[1] = \nu_2.E$, $\nu_1.N = (\sigma(!\Sigma_1.C)).arg[0] \sim^{\beta'} (\sigma(!\Sigma_1.C)).arg[0] = \nu_2.N$, $\nu_1(aL) = getAncestors(\nu_1.N, \theta_1) \sim^{\beta'} getAncestors(\nu_2.N, \theta_2) = \nu_2(aL)$, $\nu_1(\mathcal{N}) = \nu_2(\mathcal{N}) = []$, $\nu_1.\mathcal{N} = \nu_2.\mathcal{N} = []$, $\nu_1.\mathcal{H} = \nu_2.\mathcal{H} = []$ and $\nu_1.P = \nu_2.P = \mathcal{S}$. For $\nu_1.\ell$ and $\nu_2.\ell$ the following cases arise from the heap-equivalence definition:

- The path from o to root consists of only L pointers (pointer label corresponds to the $\Gamma(parentLabel)$) in both cases. In that case $\nu_1.\ell = \nu_2.\ell = L$.
- There is at least one H pointer in the path from o to the root. In that case $\nu_1.\ell \geq H$ and $\nu_2.\ell \geq H$.

Hence, $\nu_1.\ell \sim^{\beta'} \nu_2.\ell$. Therefore, $\nu_1 \sim^{\beta'} \nu_2$ from Definition 4. Thus, $\Sigma_1' \sim^{\beta'} \Sigma_2'$ by Definition 5.

6) Start-to-Capture: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. As $!\Sigma_1.aL \sim^{\beta'} !\Sigma_2.aL$, $nl_1 \sim^{\beta'} nl_2$ (Definition 2). Both Σ_1 and Σ_2 get updated in the equivalent way: $\Sigma_1' := \Sigma_1[!\Sigma_1.\mathcal{N} := nl_1, !\Sigma_1.P := \mathcal{C}]$ and $\Sigma_2' := \Sigma_2[!\Sigma_2.\mathcal{N} := nl_2, !\Sigma_2.P := \mathcal{C}]$. So, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
7) Capture-to-Target: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. For the two runs on nodes N_1 and N_2, $nl_1 = [N_1]$ and $nl_2 = [N_2]$. We know $N_1 \sim^{\beta'} N_2$, since $\Sigma_1 \sim^{\beta'} \Sigma_2$ and $!\Sigma_1.\ell = !\Sigma_2.\ell = L$, hence $nl_1 \sim^{\beta'} nl_2$. Both Σ_1 and Σ_2 get updated in the following way: $\Sigma_1' := \Sigma_1[!\Sigma_1.\mathcal{N} := nl_1, !\Sigma_1.P := \mathcal{T}]$ and $\Sigma_2' := \Sigma_2[!\Sigma_2.\mathcal{N} := nl_2, !\Sigma_2.P := \mathcal{T}]$. So, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
8) Target-to-Bubble: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. As $!\Sigma_1.aL \sim^{\beta'} !\Sigma_2.aL$, $(nl_1 := !\Sigma_1.\mathcal{N}) \sim^{\beta'} (nl_2 := !\Sigma_2.\mathcal{N})$ (Definition 2). Both Σ_1 and Σ_2 get updated in the equivalent way: $\Sigma_1' := \Sigma_1[!\Sigma_1.\mathcal{N} := nl_1, !\Sigma_1.P := \mathcal{B}]$ and $\Sigma_2' := \Sigma_2[!\Sigma_2.\mathcal{N} := nl_2, !\Sigma_2.P := \mathcal{B}]$. So, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
9) Bubble-to-Default: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. As $!\Sigma_1.aL \sim^{\beta'} !\Sigma_2.aL$, $nl_1 \sim^{\beta'} nl_2$ (Definition 2). Both Σ_1 and Σ_2 get updated in the equivalent way: $\Sigma_1' := \Sigma_1[!\Sigma_1.\mathcal{N} := nl_1, !\Sigma_1.P := \mathcal{D}]$ and $\Sigma_2' := \Sigma_2[!\Sigma_2.\mathcal{N} := nl_2, !\Sigma_2.P := \mathcal{D}]$. So, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
10) Default Prevented: $\beta' = \beta$. Only thing that changes is the phase $!\Sigma_1'.P = \Sigma_2'.P = \mathcal{D}$.
11) Remove Frame: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. Since $\Sigma_1 \sim^{\beta'} \Sigma_2$ and $!\Sigma_1(\ell) = !\Sigma_2(\ell) = L$, after popping also we will have $\Sigma_1' \sim^{\beta'} \Sigma_2'$ by Definition 5.
12) Get Event Handlers: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. Since $\mathcal{N}_1 \sim^{\beta'} \mathcal{N}_2$, therefore $\Sigma_1'.\mathcal{N} \sim^{\beta'} \Sigma_2'.\mathcal{N}$ by Definition 2. From Definition 2 either $\Gamma(n_1) = \Gamma(n_2) = L$ and $n_1 = n_2$ or $\Gamma(n_1) \geq H \wedge \Gamma(n_2) \geq H$, in either case $\Sigma_1'.\mathcal{H} \sim^{\beta'} \Sigma_2'.\mathcal{H}$ by Definition 3.
13) Run Event Handlers: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. Since its a L transition and $!\Sigma_1.\mathcal{H} \sim^{\beta'} !\Sigma_2.\mathcal{H}$, either $\Gamma(f_{m1}) = \Gamma(f_{m2}) = L \wedge f_{m1} = f_{m2}$ or $\Gamma(f_{m1}) = \Gamma(f_{m2}) = H$. Hence, $C_1 \sim^{\beta'} C_2$. Also, $!\Sigma_1'.M = !\Sigma_2'.M = R$ and $!\Sigma_1'.\mathcal{H} \sim^{\beta'} !\Sigma_2'.\mathcal{H}$. Thus, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.
14) Stop Propagation and Stop Immediate Propagation: Trivial.
15) Run Suspended Machine: $\beta' = \beta$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$. Also, $!\Sigma_1'.M = !\Sigma_2'.M = R$. Thus, $\Sigma_1' \sim^{\beta'} \Sigma_2'$.

■

Theorem 1. $\forall \kappa_1, \kappa_2. \kappa_1 \sim^{\beta} \kappa_2 \implies \kappa_1 \mathcal{R} \kappa_2$

Proof: Given $\kappa_1 \sim^{\beta} \kappa_2$. We need to prove all the clauses in the definition of $\mathcal{R}$.

1) Given
2) From Lemma 1
3) From Lemma 1
4) From Lemma 2
5) From Lemma 2
6) From Lemma 3

■

Theorem 2. Termination-insensitive non-interference
If $\kappa_1 \sim^{\beta} \kappa_2$, $\kappa_1 \twoheadrightarrow_{\alpha} \kappa_1'$ and $\kappa_2 \twoheadrightarrow_{\alpha} \kappa_2'$, then either:

- $\kappa_1' \sim^{\beta} \kappa_2'$ *or*
- $\kappa_1' \sim^{\beta} \kappa_2$ *or*
- $\kappa_1 \sim^{\beta} \kappa_2'$

Proof: Follows immediately from Theorem 1. ■

B. Formal Model for IFC in JS with DOM calls

We use the formal model from [15] and extend it to reason about the DOM API. In particular, we modify their opCall case to reason about the native and DOM calls in various lemmas. We lift the other cases of the proofs as is from their model. Below we present the complete model from [15], along with the changes to their proofs and the model.

```
struct SourceCode{
  String programSrc;
  bool strictMode;
};

typedef char* Opcode;

union Operand{
  int immediateValue;
  String identifier;
  int registerIndex;
  int funcIndex;
  bool flag;
  int offset;
};

struct Instruction{
  Opcode opc;
  Operand* opr;
};

struct CFGNode{
  Instruction* inst;
  struct CFGNode* left;
  struct CFGNode* right;
  struct CFGNode* succ;
};

struct CFG{
  struct CFGNode* cfgNode;
  JSGlobalObject* globalObject;
  int numVars;
  int numFns;
  bool strictMode;
};

struct JSLabel{
  uint64_t label;
};

enum Specials{
  NaN, undefined
};

union ValueType{
  bool b;
  int n;
  String s;
  double d;
  JSObject* o;
};

union valueTemplate{
  Specials s;
  ValueType v;
};

struct JSValue{
  valueTemplate data;
  JSLabel label;
};

struct PropertyDescriptor{
  JSValue value;
  bool writable;
  bool enumerable;
  bool configurable;
  JSLabel structLabel;
};

struct Property{
  String propertyName;
  PropertyDescriptor* pDesc;
};

struct PropertySlot{
  Property prop;
  PropertySlot* next;
};

struct JSObject{
  Property property[MAX_PROPS];
  struct proto{
    JSLabel l;
    JSObject* __proto__;
  } prototype;
  JSLabel structLabel;
};

struct Heap{
  unsigned location;
  JSObject o;
}[HEAP_SIZE];

enum FunctionType{
  JSFunction, HostFunction
};

struct JSFunctionObject:
  JSObject{
  CFG* funcCFG;
  ScopeChainNode* scopeChain;
  FunctionType fType;
};

struct JSGlobalObject:
  JSObject{
  JSFunctionObject evalFunction;
  JSObject* objectPrototype;
  JSObject* functionPrototype;
};

struct Register{
  JSValue value;
};

struct CallFrameNode{
  Register rf[REGISTER_FILE_SIZE];
  CFG cfg;
  CFGNode* returnAddress;
  ScopeChainNode* sc;
  JSFunctionObject* callee;
  JSLabel calleeLabel;
  int argCount;
  bool getter;
  int dReg;
};

struct CallFrameStack{
  CallFrameNode cFN;
  CallFrameStack* previous;
};

struct PCNode{
  JSLabel l;
  CFGNode* ipd;
  CallFrameNode* cFN;
  bool handler;
};

struct PCStack{
  PCNode node;
  PCStack* previous;
};

struct JSActivation{
  CallFrameNode* callFrameNode;
  JSLabel structLabel;
};

enum ScopeChainObjectType{
  LexicalObject, VariableObject
};

union SChainObject{
  JSObject* obj;
  JSActivation* actObj;
};

struct ScopeChainNode{
  SChainObject Object;
  ScopeChainObjectType scObjType;
  ScopeChainNode* next;
  JSLabel scopeLabel;
};
```

Figure 6: Data Structures

1) Algorithms: The different meta-functions used in the semantics presented in Section VIII-B2 are described below:

```
procedure isInstanceOf(JSLabel context, JSValue obj, JSValue protoVal)
    oProto := obj.prototype.__proto__
    while oProto do
        if oProto = protoVal then
            ret := JSValue::construct(true)
            ret.label := context
            return ret
        end if
        oProto := oProto.prototype.__proto__
        context := context.Join(oProto.prototype.l)
    end while
    ret := JSValue::construct(false)
```

```
    ret.label := context
    return ret
end procedure


procedure opRet(CallFrameStack* callStack, int ret)
    JSValue retValue = callStack.cFN.rf[ret].value
    if hostCallFrameFlag then
        callStack.pop()
        return nil, callStack, retValue
    end if
    callStack.pop()
    return callStack.cFN.returnAddress, callStack, retValue
end procedure


procedure opCall(CallFrameStack* callStack, CFGNode* ip, int func, int argCount)
    JSValue funcValue := callStack.cFN.rf[func].value
    JSFunctionObject fObj
    CallFrameNode *sigmaTop := new CallFrameNode()
    CallFrameNode *prevTop := callStack.top()
    callStack.push(sigmaTop)
    CallType callType := getCallData(funcValue, &fObj)
    if callType = CallTypeJS then
        ScopeChainNode* sc := fObj.scopeChain
        callStack.cFN.cfg := fObj.funcCFG
        callStack.cFN.returnAddress := ip.Succ
        callStack.cFN.sc := sc
        callStack.cFN.argCount := argCount
        for i ← 0, argCount do
            callStack.cFN.rf[sigmaTop.baseRegister() + i].value :=
              prevTop.rf[prevTop.headRegister()-i].value
        end for
        ip := callStack.cFN.cfg.cfgNode
    else if callType = CallTypeHost then
        ScopeChainNode* sc := fObj.scopeChain
        callStack.cFN.cfg := fObj.funcCFG
        callStack.cFN.returnAddress := ip.Succ
        callStack.cFN.sc := sc
        callStack.cFN.argCount := argCount
        functionReturnValue = invokeNativeMethod(callStack)
    end if
    retState.ip := ip
    retState.sigma := callStack
    return retState
end procedure


procedure opCallEval(JSLabel contextLabel, CallFrameStack* callStack, CFGNode* ip, int func, int argCount)
    JSValue funcValue := callStack.cFN.rf[func].value
    JSFunctionObject* fObj
    JSObject* variableObject
    Argument* arguments
    if isHostEval(funcValue) then
        ScopeChainNode* sc := fObj.scopeChain
        callStack.cFN.returnAddress := ip + 1
        callStack.cFN.sc := sc
        callStack.cFN.argCount := argCount
        SourceCode progSrc := funcValue.getSource()
        Compiler::preparse(progSrc)
        CFG* evalCodeBlock := Compiler::compile(progSrc)
        unsigned numVars := evalCodeBlock.numVariables()
        unsigned numFuncs := evalCodeBlock.numFuncDecls()
        if numVars || numFuncs then
            if evalCodeBlock.strictMode then
                JSActivation* variableObject := new JSActivation()
                variableObject.create(callStack)
                SChainObject* scObj
                scObj.actObj := variableObject
                sc.push(scObj, variableObject, contextLabel)
            else
                for (ScopeChainNode* n := sc;; n := n.next) do
                    if n.isVariableObject() && !n.isLexicalObject() then
                        variableObject := n.getObject()
                        break
                    end if
                end for
            end if
            for i ← 0, numVars do
                Identifier iden := evalCodeBlock.variable(i)
```

```
            if !variableObject.hasProperty(iden) then
                variableObject.insertVariable(iden)
            end if
        end for
        for i ← 0, numFuncs do
            JSFunctionObject* fObj := evalCodeBlock.funcDecl(i)
            variableObject.insertFunction(fObj)
        end for
    end if
    callStack.cFN.cfg := evalCodeBlock
    ip := evalCodeBlock.cfgNode
    retState.ip := ip
    retState.sigma := callStack
    return retState
  else
    return opCall(contextLabel, callStack, ip, func, argCount)
  end if
end procedure


procedure createArguments(Heap* h, CallFrameStack* callStack)
  JSObject* jsArgument := JSArgument::create(h, callStack)
  h.o[++(h.location)] := *jsArgument
  retState.theta := h
  retState.val := JSValue::construct(jsArgument)
  return retState
end procedure


procedure newFunc(CallFrameStack* callStack, Heap* heap, int funcIndex, JSLabel context)
  CFG* cBlock := callStack.cFN.cfg
  SourceCode fcCode := cBlock.getFunctionSrc(funcIndex)
  CFG* fcBlock := Compiler::compile(fcCode,callStack.cFN)
  JSFunctionObject* fObj := JSFunctionObject::create( fcBlock,callStack.cFN.sc)
  fObj.structLabel := context
  heap.o[++(heap.location)] := *fObj
  retState.theta := heap
  retState.val := JSValue::construct(fObj)
  return retState
end procedure


procedure createActivation(CallFrameStack* callStack, JSLabel contextLabel)
  JSActivation* jsActivation := new JSActivation()
  jsActivation.create(callStack)
  jsActivation.structLabel := contextLabel
  SChainObject* scObj
  scObj.actObj := jsActivation
  JSValue vActivation := JSValue::jsValuefromActivation (jsActivation)
  if callStack.cFN.scopeLabel ≥ contextLabel then
    callStack.cFN.sc.push(scObj, VariableObject, contextLabel)
    callStack.cFN.scopeLabel := contextLabel
  else
    stop
  end if
  return retState
end procedure


procedure createThis(JSLabel contextLabel, CallFrameStack* callStack, Heap* h)
  JSFunctionObject* callee := callStack.cFN.callee
  PropertySlot p(callee)
  String str := "prototype"
  JSValue proto := p.getValue(str)
  JSObject* obj := new JSObject()
  obj.structLabel := contextLabel
  obj.prototype.__proto__ := proto.toObject()
  obj.prototype.l := proto.toObject().structLabel.join( contextLabel)
  h.o[++(h.location)] := *obj
  retState.theta := h
  retState.val := JSValue::construct(obj)
  return retState
end procedure


procedure newObject(Heap* h, JSLabel contextLabel)
  JSObject* obj := emptyObject()
  obj.structLabel := contextLabel
  obj.prototype.__proto__ := ObjectPrototype::create()
  obj.prototype.l := contextLabel
```

```
    h.o[++(h.location)] := *obj
    retState.theta := h
    retState.val := JSValue::construct(obj)
    return retState
end procedure


procedure getPropertyById(JSValue v, String p, int dst)
    JSObject* O := v.toObject()
    JSLabel label := O.structLabel
    JSValue ret := jsUndefined()
    if O.isUndefined() then
        ret.label := label
        return ret
    end if
    while O ≠ null do
        if O.containsProperty(p) then
            if p.isGetter() then
                JSValue v = p.getValue()
                JSFunctionObject* funcObj = (JSFunctionObject*) v.toObject()
                CallFrameNode *sigmaTop = new CallFrameNode()
                callStack.push(sigmaTop)
                ScopeChainNode* sc = fObj.scopeChain
                CFG* newCodeBlock = fObj.funcCFG
                callStack.cFN.cfg = *newCodeBlock
                callStack.cFN.returnAddress = ip + 1
                callStack.cFN.sc = sc
                callStack.cFN.getter = true
                callStack.cFN.dReg = dst
                ip = newCodeBlock.cfgNode
                interpreter.iota = ip
                interpreter.sigma = callStack
            else
                ret := getProperty(p).getValue()
                ret.label := label
            end if
            return ret
        else
            O := O.prototype.__proto__
        end if
        label := label.join(label)
    end while
end procedure


procedure putDirect(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int base, String property, int propVal)
    JSValue baseValue := callStack.cFN.rf[base]. value
    JSValue propValue := callStack.cFN.rf[propVal]. value
    JSObject* obj := baseValue.toObject()
    PropertyDescriptor dataPD := PropertyDescriptor::createPD(true, true, true)
    dataPD.value := propValue
    obj.setProperty(property, dataPD)
    obj.structLabel := obj.structLabel.join(contextLabel)
    h.o[++(h.location)] := *obj
    return h
end procedure


procedure putIndirect(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int base, String property, int val)
    JSValue baseValue := callStack.cFN.rf[base].value
    JSValue propValue := callStack.cFN.rf[val].value
    JSObject* obj := baseValue.toObject()
    bool isStrict := callStack.cFN.cfg.isStrictMode()
    contextLabel := obj.structLabel.join(contextLabel)
    if obj.containsPropertyInItself(property) && obj. getProperty(property).isDataProperty() && !isStrict && obj.isWritable() then
        obj.getProperty(property).setValue(propValue)
        h.o[++(h.location)] := *obj
        return h
    end if
    return putDirect(contextLabel, callStack, h, base, property, val)
end procedure


procedure delById(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int base, Identifier property)
    JSValue baseValue := callStack.cFN.rf[base].value
    JSObject* obj := baseValue.toObject()
    int loc := h.findObject(obj)
    Property prop := obj.getProperty(property)
    PropertyDescriptor pd := prop.getPropertyDescriptor()
    if obj.getPropertyValue(prop).label ≥ contextLabel then
```

```
        if !obj.containsPropertyInItself(property) then
            retState.theta := h
            retState.val := JSValue::construct(true)
            return retState
        end if
        if obj.containsPropertyInItself(property) && prop. isConfigurable() then
            if !(callStack.cFN.cfg.isStrictMode()) then
                pd.value := JSValue::constructUndefined()
                obj.setProperty(property, pd)
                h.o[loc] := *obj
                retState.theta := h
                retState.val := JSValue::construct(true)
                return retState
            end if
        end if
        retState.theta := h
        retState.val := JSValue::construct(false)
        return retState
    else
        stop
    end if
end procedure


procedure putGetterSetter(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int base, Identifier property, JSValue getterValue, JSValue setterValue)
    JSValue baseValue := callStack.cFN.rf[base].value
    JSObject* obj := baseValue.toObject()
    int loc := h.findObject(obj)
    JSFunctionObject *getterObj, *setterObj
    JSFunctionObject *getterFuncObj := null, *setterFuncObj := null
    if !getterValue.isUndefined() then
        getterFuncObj := getterValue.toFunctionObject (callStack.cFN.cfg, callStack.cFN.sc)
    end if
    if !setterValue.isUndefined() then
        setterFuncObj := setterValue.toFunctionObject (callStack.cFN.cfg, callStack.cFN.sc)
    end if
    if getterFuncObj ≠ null then
        obj.setGetter(property, getterObj)
    end if
    if setterFuncObj ≠ null then
        obj.setSetter(setterObj)
    end if
    PropertyDescriptor accessor := PropertyDescriptor ::createPD(false, false, false, true)
    JSValue v := JSValue::constructUndefined()
    v.label := contextLabel
    accessor.value := v
    obj.setProperty(property, accessor)
    obj.structLabel := contextLabel
    h.o[loc] := *obj
    return h
end procedure


procedure getPropNames(CallFrameStack* callStack, Instruction* ip, int base, int i, int size, int breakOffset)
    JSValue baseVal := callStack.cFN.rf[base].value
    JSObject* obj := baseVal.toObject()
    PropertyIterator* propItr := obj.getProperties()
    if baseVal.isUndefined() || baseVal.isNull() then
        retState.v1 := jsUndefined()
        retState.v2 := jsUndefined()
        retState.v3 := jsUndefined()
        retState.ip := ip + breakOffset
        return retState
    end if
    retState.v1 := JSValue::construct(propItr)
    retState.v2 := JSValue::construct(0)
    retState.v3 := JSValue::construct(propItr.size())
    retState.ip := ip + 1
    return retState
end procedure


procedure getNextPropName(CallFrameStack* cStack, Instruction* ip, JSValue base, int i, int size, int iter, int offset, int dst)
    JSObject* obj := base.toObject()
    PropertyIterator* propItr := cStack.cFN.rf[iter].value. toPropertyIterator()
    int b := rFile[i].value.toInteger()
    int e := rFile[size].value.toInteger()
    while b < e do
        String key := propItr.get(b)
        retState.value1 := JSValue::construct(b + 1)
```

```
        if !(key.isNull()) then
            retState.value2 := JSValue::construct(key)
            ip := ip + offset
            break
        end if
        b++
    end while
    return retState
end procedure


procedure resolveInSc(JSLabel contextLabel, ScopeChainNode* scopeHead, String property)
    JSValue v
    JSLabel l
    ScopeChainNode* scn := scopeHead
    while scn ≠ NULL do
        PropertySlot pSlot := scn.getPropertySlot()
        if pSlot.contains(property) then
            v := pSlot.getValue(property)
            v.label := contextLabel
            return v
        end if
        scn := scn.next
        if scn.scObjType = VariableObject then
            contextLabel = contextLabel.join(scn.Object. actObj.structLabel)
        else if scn.scObjType = LexicalObject then
            contextLabel = contextLabel.join(scn.Object. obj.structLabel)
        end if
        contextLabel := contextLabel.join(scn. scopeNextLabel)
    end while
    v := jsUndefined()
    v.label := contextLabel
    return v
end procedure


procedure resolveInScWithSkip(JSLabel contextLabel, ScopeChainNode* scopeHead, String property, int skip)
    JSValue v
    JSLabel l
    ScopeChainNode* scn := scopeHead
    while skip−− do
        scn := scn.next
        if scn.scObjType = VariableObject then
            contextLabel := contextLabel.join( scn.Object. actObj.structLabel)
        else if scn.scObjType = LexicalObject then
            contextLabel := contextLabel.join(scn.Object. obj.structLabel)
        end if
        contextLabel := contextLabel.join(scn. scopeNextLabel)
    end while
    while scn ≠ null do
        PropertySlot pSlot := scn.getPropertySlot()
        if pSlot.contains(property) then
            v := pSlot.getValue(property)
            v.label := contextLabel
            return v
        end if
        scn := scn.next
        if scn.scObjType = VariableObject then
            contextLabel := contextLabel.join(scn.Object. actObj.structLabel)
        else if scn.scObjType = LexicalObject then
            contextLabel := contextLabel.join(scn.Object. obj.structLabel)
        end if
        contextLabel := contextLabel.join(scn. scopeNextLabel)
    end while
    v := jsUndefined()
    v.label := contextLabel
    return v
end procedure


procedure resolveGlobal(JSLabel contextLabel, CallFrameStack* cStack, String property)
    JSValue v
    struct CFG* cBlock := cStack.cFN.cfg
    JSGlobalObject* globalObject := cBlock. getGlobalObject()
    PropertySlot pSlot(globalObject)
    if pSlot.contains(property) then
        v := pSlot.getValue(property)
        v.label := contextLabel
        return v
    end if
```

```
    v := jsUndefined()
    v.label := contextLabel
    return v
end procedure


procedure resolveBase(JSLabel contextLabel, CallFrameStack* cStack, ScopeChainNode* scopeHead, String property, bool strict)
    JSValue v
    ScopeChainNode* scn := scopeHead
    CFG *cBlock := cStack.cFN.cfg
    JSGlobalObject *gObject := cBlock.globalObject
    while scn ≠ null do
        JSObject* obj := scn.get()
        contextLabel := obj.structLabel.join(contextLabel)
        PropertySlot pSlot(obj)
        if scn.next = null && strict && !pSlot.contains (property) then
            v := emptyJSValue()
            v.label := contextLabel
            return v
        end if
        if pSlot.contains(property) then
            v := JSValueContainingObject(obj)
            v.label := contextLabel
            return v
        end if
        scn := scn.next
        if scn ≠ null then
            contextLabel := contextLabel.join(scn. scopeNextLabel)
        end if
    end while
    v := JSValue::construct(gObject)
    v.label := contextLabel
    return v
end procedure


procedure resolveBaseAndProperty(JSLabel contextLabel, CallFrameStack cStack, int bRegister, int pRegister, String property)
    JSValue v
    ScopeChainNode* scn := cStack.cFN.sc
    while scn ≠ null do
        JSObject* obj := scn.get()
        contextLabel := obj.structLabel.join(contextLabel)
        PropertySlot pSlot(obj)
        if pSlot.contains(property) then
            v := pSlot.getValue(property)
            v.label := contextLabel
            ret.val1 := v
            v := JSValueContainingObject(obj)
            v.label := contextLabel
            ret.val2 := v
            return ret
        end if
        scn := scn.next
        if scn ≠ null then
            contextLabel := contextLabel.join(scn. scopeNextLabel)
        end if
    end while
end procedure


procedure getScopedVar(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int index, int skip)
    JSValue v
    ScopeChainNode* scn := callStack.cFN.sc
    while skip−− do
        if scn.scObjType = VariableObject then
            contextLabel := contextLabel.join(scn.Object.actObj. structLabel)
        else if scn.scObjType = LexicalObject then
            contextLabel := contextLabel.join(scn.Object.obj. structLabel)
        end if
        contextLabel := contextLabel.join(scn. scopeLabel)
        scn := scn.next
    end while
    v := scn.registerAt(index)
    if scn.scObjType = VariableObject then
        v.label := contextLabel.join(scn.Object.actObj. structLabel)
    else if scn.scObjType = LexicalObject then
        v.label := contextLabel.join(scn.Object.obj. structLabel)
    end if
    return v
end procedure
```

```
procedure putScopedVar(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int index, int skip, int value)
    CallFrameStack* cStack
    ScopeChainNode* scn := callStack.cFN.sc
    JSValue val := callStack.cFN.rf[value].value
    while skip−− do
        if scn.scObjType = VariableObject then
            contextLabel := contextLabel.join(scn.Object. actObj.structLabel)
        else if scn.scObjType = LexicalObject then
            contextLabel := contextLabel.join(scn.Object. obj.structLabel)
        end if
        contextLabel := contextLabel.join(scn. scopeLabel)
        scn := scn.next
    end while
    cStack := scn.setRegisterAt(contextLabel, index, val)
    return cStack
end procedure


procedure pushScope(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int scope)
    ScopeChainNode* sc := callStack.cFN.sc
    JSValue v := callStack.cFN.rf[scope].value
    JSObject* o := v.toObject()
    SChainObject* scObj
    if sc.scopeLabel ≥ contextLabel then
        scObj.obj := o
        sc.push(scObj, LexicalObject, contextLabel)
        callStack.cFN.sc := sc
    else if sc.scopeLabel = star then
        scObj.obj := o
        sc.push(scObj, LexicalObject, star)
        callStack.cFN.sc := sc
    end if
    return callStack
end procedure


procedure popScope(JSLabel contextLabel, CallFrameStack* callStack, Heap* h)
    ScopeChainNode* sc := callStack.cFN.sc
    JSLabel l := sc.scopeLabel
    if l ≥ contextLabel then
        sc.pop()
        callStack.cFN.sc := sc
    else
        stop
    end if
    return callStack
end procedure


procedure jmpScope(JSLabel contextLabel, CallFrameStack* callStack, Heap* h, int count)
    ScopeChainNode* sc := callStack.cFN.sc
    while count−− > 0 do
        JSLabel l := sc.scopeLabel
        if l ≥ contextLabel then
            sc.pop()
            callStack.cFN.sc := sc
        else
            stop
        end if
    end while
    return callStack
end procedure


procedure throwException(CallFrameStack* callStack, CFGNode* iota)
    CFGNode* handler
    while callStack.cFN.hasHandler()==false do
        callStack.pop()
    end while
    while callStack.cFN.sc.length() - callStack.cFN.getHandlerScopeDepth() do
        callStack.cFN.sc.pop()
    end while
    handler := callStack.cFN.getHandler(iota)
    interpreter.iota := handler
    interpreter.sigma := callStack
end procedure
```

2) Semantics:

$$\text{prim:}\ \frac{\begin{array}{c}\iota = \text{“prim dst:r src1:r src2:r”} \\ \mathcal{L} := \Gamma(!\sigma(src1)) \sqcup \Gamma(!\sigma(src2)) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(!\sigma(src1)) \oplus \Upsilon(!\sigma(src2)) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

prim reads the values from two registers ($src1$ and $src2$), performs the binary operation generically denoted by $\oplus$, and writes the result into the dst register. The label assigned to the value in dst register is the join of the label of value in $src1$, $src2$ and the head of the pc-stack ($!\rho$). In order to avoid implicit leak of information, the label of the existing value in dst is compared with the current context label. If the label is lower than the context label, the label of the value in dst is set to $\star$.

$$\text{mov:}\ \frac{\begin{array}{c}\iota = \text{“mov dst:r src:r”} \\ \mathcal{L} := \Gamma(!\sigma(src)) \sqcup \Gamma(!\rho) \qquad \mathcal{V} = \Upsilon(!\sigma(src)) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

mov copies the value from the src register to the dst register. The label assigned to the value in dst register is the join of the label of value in src and the head of the pc-stack ($!\rho$). In order to avoid implicit leak of information, the label of the existing value in dst is compared with the current context label. If the label is lower than the context label, the label of the value in dst is joined with $\star$.

$$\text{jfalse:}\ \frac{\begin{array}{c}\iota = \text{“jfalse cond:r target:offset”} \\ \Gamma(!\sigma(cond)) \neq \star \qquad \rho'' := \rho.push(\Gamma(!\sigma(cond)) \sqcup \Gamma(!\rho), IPD(\iota), CF(\iota), false) \\ \Upsilon(!\sigma(cond)) = false \Rightarrow \iota' := Left(!\sigma.CFG, \iota) \diamond \iota' := Right(!\sigma.CFG, \iota) \\ \rho' := isIPD(\iota', \rho'', \sigma)\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma, \rho'}$$

jfalse is a branching instruction. Based on the value in the $cond$ register, it decides which branch to take. The operation is performed only if the value in $cond$ is not labelled with a $\star$. If it contains a $\star$, we terminate the execution to prevent possible leak of information. The push function defined in the rule does the following: A node is pushed on the top of the pc-stack containing the IPD of the branching instruction and the label of the value in $cond$ joined with the context, to define the context of this branch. If the IPD of the instruction is SEN or the same as the top of the pc-stack, then we just join the label on top of the pc-stack with the context label determined by the $cond$ register.

$$\text{loop-if-less:}\ \frac{\begin{array}{c}\iota = \text{“loop-if-less src1:r src2:r target:offset”} \\ \Gamma(!\sigma(src1)) \neq \star \qquad \Gamma(!\sigma(src2)) \neq \star \qquad \mathcal{L} := \Gamma(!\sigma(src1)) \sqcup \Gamma(!\sigma(src2)) \sqcup \Gamma(!\rho) \\ \Upsilon(!\sigma(src1)) < \Upsilon(!\sigma(src2)) \Rightarrow \iota' := Left(!\sigma.CFG, \iota) \diamond \iota' := Right(!\sigma.CFG, \iota) \\ \rho'' := \rho.push(\mathcal{L}, IPD(\iota), CF(\iota), false) \qquad \rho' := isIPD(\iota', \rho'', \sigma)\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma, \rho'}$$

loop-if-less is another branching instruction. If the value of $src1$ is less than $src2$, then it jumps to the *target*, else continues with the next instruction. The operation is performed only if the values in $src1$ and $src2$ are not labelled with a $\star$. If any one of them contains a $\star$, we abort the execution to prevent possible leak of information. The push function defined in the rule does the following: A node is pushed on the top of the pc-stack containing the IPD of the branching instruction and the join of the label of the values in $src1$ and $src2$ joined with the context, to define the context of this branch. If the IPD of the instruction is SEN or the same as the top of the pc-stack, then we just join the label on top of the pc-stack with the context label determined above.

$$\text{typeof:}\ \frac{\begin{array}{c}\iota = \text{“typeof dst:r src:r”} \\ \mathcal{L} := (\Gamma(src) \sqcup \Gamma(!\rho)) \qquad \mathcal{V} := determineType(!\sigma(src)) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

typeof determines the type string for src according to ECMAScript rules, and puts the result in register dst. We do a deferred NSU check on dst before writing the result in it. The *determineType* function returns the data type of the value passed as the parameter.

$$\text{instanceof:}\ \frac{\begin{array}{c}\iota = \text{“instanceof dst:r value:r cProt:r”} \\ v := isInstanceOf(\Gamma(!\rho), !\sigma(value), cProt) \qquad \mathcal{L} = \Gamma(v) \qquad \mathcal{V} = \Upsilon(v), \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star), \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

instanceof tests whether the *cProt* is in the prototype chain of the object in register *value* and puts the Boolean result in the dst register after deferred NSU check.

$$\text{enter:}\ \frac{\iota = \text{“enter”} \qquad \iota' := Succ(!\sigma.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma)}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma, \rho'}$$

enter marks the beginning of a code block.

$$\text{ret:}\ \frac{\iota = \text{“ret res:r”} \qquad (\iota', \sigma', \gamma) := opRet(\sigma, res) \qquad \rho' := isIPD(\iota', \rho, \sigma')}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

ret is the last instruction to be executed in a function. It pops the call-frame and returns the control to the callee's call-frame. The return value of the function is written to a local variable in the interpreter (γ), which can be read by the next instruction being executed.

$$\text{end:}\ \frac{\iota = \text{“end res:r”} \qquad opEnd(\sigma, res)}{\theta, \iota, \sigma, \rho \ \rightarrow\ -}$$

end marks the end of a program. *opEnd* passes the value present in *res* register to the caller (the native function that invoked the interpreter).

$$\text{call:}\ \frac{\begin{array}{c}\iota = \text{“op-call func:r args:n”} \\ \Gamma(func) \neq \star \qquad (\iota', \sigma', \mathcal{H}, \ell_f) := opCall(\sigma, \iota, func, args) \\ \rho'' := \rho.push(\ell_f \sqcup \Gamma(!\sigma(func)) \sqcup \Gamma(!\rho), IPD(\iota), CF(\iota), \mathcal{H}) \qquad \rho' := isIPD(\iota', \rho'', \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \iota', \theta, \sigma', \rho'}$$

call, initially, checks the function object's label for $\star$ and if the label contains a $\star$, the program execution is aborted. The reason for termination is the possible leak of information as explained above. If not, call creates a new call-frame, copies the arguments, initializes the registers, scope-chain pointer, codeblock and the return address. The registers are initialized to *undefined* and assigned a label obtained by joining the label of the context in which the function was created and the label of the function object itself. We treat *call* as a branching instruction and hence, push a new node on the top of the pc-stack with the label determined above along with its IPD and call-frame. The field $\mathcal{H}$ in the push function is determined by looking up the exception table. If it contains an associated exception handler, it sets the field to $true$ else it is set to $false$. If the IPD is the SEN then we just join the label on the top of the stack with the currently calculated label. It then points the instruction pointer to the first instruction of the new code block.

$$\text{call-put-result:}\ \frac{\begin{array}{c}\iota = \text{“call-put-result res:r”} \\ \mathcal{L} := \Gamma(\gamma) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(\gamma) \\ (\Gamma(!\sigma(res)) \geq \Gamma(!\rho)) \Rightarrow \mathcal{L} := \mathcal{L} \diamond \mathcal{L} := \star \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(res)) := \mathcal{V} \\ \Gamma(!\sigma(res)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

call-put-result copies the return value γ to the *res* register. The label assigned to the value in *res* register is the join of the label of the return value and the head of the pc-stack. In order to avoid implicit leak of information, *deferred no-sensitive-upgrade* is performed.

$$\text{call-eval:}\ \frac{\begin{array}{c}\iota = \text{“call-eval func:r args:n”} \\ \Gamma(!\sigma(func)) \neq \star \qquad (\iota', \sigma', \mathcal{H}, \ell_f) := opCallEval(\Gamma(!\rho), \sigma, \iota, func, args) \\ \rho'' := \rho.push(\ell_f \sqcup \Gamma(!\sigma(func)) \sqcup \Gamma(!\rho), IPD(\iota), CF(\iota), \mathcal{H}) \qquad \rho' := isIPD(\iota', \rho'', \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \iota', \theta, \sigma', \rho'}$$

call-eval calls a function with the string passed as an argument converted to a code block. If *func* register contains the original global eval function, then it is performed in local scope, else it is similar to *call*.

$$\text{create-arguments:}\ \frac{\begin{array}{c}\iota = \text{“create-arguments dst:r”} \\ (\theta', v) := createArguments(\theta, \sigma) \qquad \mathcal{L} := \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow \mathcal{L} := \mathcal{L} \diamond \mathcal{L} := \star \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma', \rho'}$$

create-arguments creates the arguments object and places its pointer in the local dst register after the deferred NSU check. The label of the arguments object is set to the context.

$$\text{new-func:}\ \frac{\begin{array}{c}\iota = \text{“new-func dst:r funcIndex:f”} \\ (\theta', v) := newFunc(\sigma, \theta, funcIndex, \Gamma(!\rho)) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow \mathcal{L} := \mathcal{L} \diamond \mathcal{L} := \star \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma', \rho'}$$

new-func constructs a new function instance from function at *funcIndex* and the current scope chain and puts the result in dst after deferred NSU check.

$$\text{create-activation:}\ \frac{\begin{array}{c}\iota = \text{“create-activation dst:r”} \\ (\sigma', v) := createActivation(\sigma, \Gamma(!\rho)) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow \mathcal{L} := \mathcal{L} \diamond \mathcal{L} := \star \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma''(dst)):=\mathcal{V} \\ \Gamma(!\sigma''(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

create-activation creates the activation object for the current call-frame if it has not been already created and writes it to the dst after the deferred NSU check and pushes the object in the scope-chain. If the label of the head of the existing scope-chain is less than the context, then the label of the pushed node is set to $\star$, else it is set to the context.

$$\text{construct:}\ \frac{\begin{array}{c}\iota = \text{“construct func:r args:n”} \\ \Gamma(!\sigma(func)) \neq \star \qquad (\iota', \sigma', \mathcal{H}, \ell_f) := opCall(\sigma, \iota, func, args) \\ \rho'' := \rho.push(\ell_f \sqcup \Gamma(!\sigma'(func)) \sqcup \Gamma(!\rho), IPD(\iota), CF(\iota), \mathcal{H}) \qquad \rho' := isIPD(\iota', \rho'', \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \iota', \theta, \sigma', \rho'}$$

construct invokes register *func* as a constructor and is similar to *call*. For JavaScript functions, the *this* object being passed (the first argument in the list of arguments) is a new object. For host constructors, no *this* is passed.

$$\text{create-this:}\ \frac{\begin{array}{c}\iota = \text{“create-this dst:r”} \\ (\theta', v) := createThis(\Gamma(!\rho), \sigma, \theta) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow \mathcal{L} := \mathcal{L} \diamond \mathcal{L} := \star \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma', \rho'}$$

create-this creates and allocates an object as *this* used for construction later in the function. The object is labelled the context and placed in dst after deferred NSU check. The prototype chain pointer is also labelled with the context label.

$$\text{new-object:}\ \frac{\begin{array}{c}\iota = \text{“new-object dst:r”} \\ (\theta', v) := newObject(\theta, \Gamma(!\rho)) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma', \rho'}$$

new-object constructs a new empty object instance and puts it in dst after deferred NSU check. The object is labelled with the context label and the prototype chain pointer is also labelled with the context.

$$\text{get-by-id:}\ \frac{\begin{array}{c}\iota = \text{“get-by-id dst:r base:r prop:id vdst:r”} \\ v := getPropertyById(!\sigma(base), prop, vdst) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)):=\mathcal{V} \\ \Gamma(!\sigma(dst)):=\mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

get-by-id gets the property named by the identifier *prop* from the object in the *base* register and puts it into the dst register after the deferred NSU check. If the object does not contain the property, it looks up the prototype chain to determine if any of the proto objects contain the property. When traversing the prototype chain, the context is joined with the structure label of all the objects and the prototype chain pointer labels until the property is found or the end of the chain. It then joins the property label to the context. If the property is not found, it returns *undefined*. The joined label of the context is the label of the property put in the dst register.

If the property is an accessor property, it calls the getter function, sets the getter flag in the call-frame and updates the destination register field with the register where the value is to be inserted. It then transfers the control to the first instruction in the getter function.

$$\text{put-by-id:}\ \frac{\begin{array}{c}\iota = \text{“put-by-id base:r prop:id value:r direct:b”} \\ \Gamma(!\sigma(value)) \neq \star \\ (direct = true \Rightarrow \theta' := putDirect(\Gamma(!\rho), \sigma, \theta, base, prop, value) \diamond \\ \theta' := putIndirect(\Gamma(!\rho), \sigma, \theta, base, prop, value)) \\ \iota' := Succ(!\sigma.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma)\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma, \rho'}$$

put-by-id writes into the heap the property of an object. We check for $\star$ in the label of *value* register. If it contains a $\star$, the program aborts as this could potentially result in an implicit information flow. If not, it writes the property into the object. The basic functionality is to search for the property in the object and its prototype chain, and change it. If the property is not found, a new property for the current object with the property label as the context is created. Based on whether the property is in the object itself (or needs to be created in the object itself) or in the prototype chain of the object, it calls *putDirect* and *putIndirect*, respectively.

$$\text{del-by-id:}\ \frac{\begin{array}{c}\iota = \text{“del-by-id dst:r base:r prop:id”} \\ \Gamma(!\sigma(base)) \neq \star \qquad (\theta', v) := delById(\Gamma(!\rho), \sigma, \theta, base, prop) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \\ \mathcal{V} := \Upsilon(v) \qquad (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma \left[\begin{smallmatrix} \Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L} \end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma', \rho'}$$

del-by-id deletes the property specified by *prop* in the object contained in *base*. If the structure label of the object is less than the context, the deletion does not happen. If the property is found, the property is deleted and Boolean value *true* is written to dst, else it writes *false* to dst. The label of the Boolean value is the structure label of the object joined with the property label.

$$\text{getter-setter:}\ \frac{\begin{array}{c}\iota = \text{“put-getter-setter base:r prop:id getter:r setter:r”}, \\ \star \notin \Gamma(!\sigma(getter)), \quad \star \notin \Gamma(!\sigma(setter)), \\ \theta' := putGetterSetter(\Gamma(!\rho), \sigma, base, prop, !\sigma(getter), !\sigma(setter)), \\ \iota' := Succ(!\sigma.CFG, \iota), \ \ \rho' := isIPD(\iota', \rho)\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota'', \sigma, \rho'}$$

getter-setter puts the accessor descriptor to the object in register *base*. It initially checks if the structure label of the object is greater or equal to the context. The property for which the accessor properties are added is given in the register *prop*. The property label of the accessor functions is set to the context. *putGetterSetter* calls *putIndirect* internally and sets the getter/setter property of the object with the specified value.

$$\text{get-pnames:}\ \frac{\begin{array}{c}\iota = \text{“get-pnames dst:r base:r i:r size:r breakTarget:offset”} \\ \Gamma(!\sigma(base)) \neq \star \qquad (v_1, v_2, v_3, \iota') := getPropNames(\sigma, \iota, base, i, size, breakTarget) \\ \mathcal{L}_n := \Gamma(!\sigma(base)) \sqcup \Gamma(v_n) \sqcup \Gamma(!\rho) \qquad \mathcal{V}_n := \Upsilon(v_n), n := 1, 2, 3 \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}_1 := \mathcal{L}_1) \diamond (\mathcal{L}_1 := \star) \\ (\Gamma(!\sigma(i)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}_2 := \mathcal{L}_2) \diamond (\mathcal{L}_2 := \star) \\ (\Gamma(!\sigma(size)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}_3 := \mathcal{L}_3) \diamond (\mathcal{L}_3 := \star) \\ \sigma'' := \sigma \left[\begin{smallmatrix} \Upsilon(!\sigma(dst)) := \mathcal{V}_1 \\ \Gamma(!\sigma(dst)) := \mathcal{L}_1 \end{smallmatrix}\right] \qquad \sigma''' := \sigma'' \left[\begin{smallmatrix} \Upsilon(!\sigma''(i)) := \mathcal{V}_2 \\ \Gamma(!\sigma''(i)) := \mathcal{L}_2 \end{smallmatrix}\right] \qquad \sigma' := \sigma''' \left[\begin{smallmatrix} \Upsilon(!\sigma'''(size)) := \mathcal{V}_3 \\ \Gamma(!\sigma'''(size)) := \mathcal{L}_3 \end{smallmatrix}\right] \\ v_n = undefined \Rightarrow (\mathcal{L} = \Gamma(!\sigma(base))) \quad \diamond \\ (\mathcal{L} = \Gamma(!\sigma(base)) \sqcup \Gamma(\theta(!\sigma(base))) \sqcup (\forall p \in Prop(\theta(!\sigma(base))).\Gamma(p))) \\ \rho'' = \rho.push(\mathcal{L}, IPD(\iota), CF(\iota), false) \qquad \rho' := isIPD(\iota', \rho'', \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

get-pnames creates a property name list for object in register *base* and puts it in dst, initializing i and *size* for iteration through the list, after the deferred NSU check. If *base* is *undefined* or *null*, it jumps to *breakTarget*. It is a branching instruction and pushes the label with join of all the property labels and the structure label of the object along with the IPD on the pc-stack. If the IPD of the instruction is SEN or the same as the top of the pc-stack, then we just join the label on top of the pc-stack with the context label determined above.

$$\text{next-pname:}\ \frac{\begin{array}{c}\iota = \text{“next-pname dst:r base:r i:n size:n iter:n target:offset”} \\ (v_1, v_2, \iota') := getNextPropNames(\sigma, \iota, base, i, size, iter, target) \\ \mathcal{L}_n := \Gamma(v_n) \sqcup \Gamma(!\rho) \qquad \mathcal{V}_n := \Upsilon(v_n), \ n := 1, 2 \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}_1 := \mathcal{L}_1) \diamond (\mathcal{L}_1 := \star) \\ (\Gamma(!\sigma(i)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}_2 := \mathcal{L}_2) \diamond (\mathcal{L}_2 := \star) \\ \sigma'' := \sigma \left[\begin{smallmatrix} \Upsilon(!\sigma(dst)) := \mathcal{V}_1 \\ \Gamma(!\sigma(dst)) := \mathcal{L}_1 \end{smallmatrix}\right] \qquad \sigma' := \sigma'' \left[\begin{smallmatrix} \Upsilon(!\sigma''(i)) := \mathcal{V}_2 \\ \Gamma(!\sigma''(i)) := \mathcal{L}_2 \end{smallmatrix}\right] \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

next-pname copies the next name from the property name list created by *get-pnames* in *iter* to dst after deferred NSU check, and jumps to *target*. If there are no names left, it continues with the next instruction. Although, it behaves as a

branching instruction, the context pertaining to this opcode is already pushed in *get-pnames*. Also, the IPD corresponding to this instruction, is the same as the one determined by *get-pnames*. Thus, we do not push on the pc-stack in this instruction.

$$\text{resolve:}\ \frac{\begin{array}{c}\iota = \text{“resolve dst:r prop:id”} \\ v := resolveInSc(\Gamma(!\rho), !\sigma.sc, prop) \\ \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \qquad (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

resolve searches for the property in the scope chain and writes it into dst register, if found. The label of the property written in dst is a join of the context label, all the nodes (structure label of the object contained in it) traversed in the scope chain and the label associated with the pointers in the chain until the node (object) where the property is found. If the initial label of the value contained in dst was lower than the context label, then the label of the value in dst is joined with $\star$. In case the property is not found, the instruction throws an exception (similar to throw, as described later).

$$\text{resolve-skip:}\ \frac{\begin{array}{c}\iota = \text{“resolve-skip dst:r prop:id skip:n”} \\ v := resolveInScWithSkip(\Gamma(!\rho), !\sigma.sc, prop, skip) \\ \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \qquad (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

resolve-skip looks up the property named by *prop* in the scope chain similar to *resolve*, but it skips the top *skip* levels and writes the result to register dst. If the property is not found, it also raises an exception and behaves similarly to *resolve*.

$$\text{resolve-global:}\ \frac{\begin{array}{c}\iota = \text{“resolve-global dst:r prop:id”} \\ v := resolveGlobal(\Gamma(!\rho), \sigma, prop) \\ \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \qquad (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

resolve-global looks up the property named by *prop* in the global object. If the structure of the global object matches the one passed here, it looks into the global object. Else, it falls back to perform a full *resolve*.

$$\text{resolve-base:}\ \frac{\begin{array}{c}\iota = \text{“resolve-base dst:r prop:id isStrict:bool”} \\ v := resolveBase(\Gamma(!\rho), \sigma, !\sigma.sc, prop, isStrict) \\ \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \qquad (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

resolve-base looks up the property named by *prop* in the scope chain similar to *resolve* but writes the object to register dst. If the property is not found and *isStrict* is *false*, the global object is stored in dst.

$$\text{resolve-with-base:}\ \frac{\begin{array}{c}\iota = \text{“resolve-with-base bDst:r pDst:r prop:id”} \\ (bdst, pdst) := resolveBaseAndProperty(\Gamma(!\rho), \sigma, baseDst, propDst, prop) \\ \mathcal{L}1 := \Gamma(!\sigma(bdst)) \sqcup \Gamma(!\rho) \qquad \mathcal{V}1 := \Upsilon(!\sigma(bdst)) \\ \mathcal{L}2 := \Gamma(!\sigma(pdst)) \sqcup \Gamma(!\rho) \qquad \mathcal{V}2 := \Upsilon(!\sigma(pdst)) \\ (\Gamma(!\sigma(bDst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}1 := \mathcal{L}1) \diamond (\mathcal{L}1 := \star) \\ (\Gamma(!\sigma(pDst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L}2 := \mathcal{L}2) \diamond (\mathcal{L}2 := \star) \\ \sigma'' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(bDst)) := \mathcal{V}1 \\ \Gamma(!\sigma(bDst)) := \mathcal{L}1\end{smallmatrix}\right] \qquad \sigma' := \sigma''\left[\begin{smallmatrix}\Upsilon(!\sigma''(pDst)) := \mathcal{V}2 \\ \Gamma(!\sigma''(pDst)) := \mathcal{L}2\end{smallmatrix}\right] \\ \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

resolve-with-base looks up the property named by *prop* in the scope chain similar to *resolve-base* and writes the object to register *bDst*. It also, writes the property to *pDst*. If the property is not found it raises an exception like *resolve*.

$$\text{get-scoped-var:}\ \frac{\begin{array}{c}\iota = \text{“get-scoped-var dst:r index:n skip:n”} \\ v := getScopedVar(\Gamma(!\rho), \sigma, \theta, index, skip) \qquad \mathcal{L} := \Gamma(v) \sqcup \Gamma(!\rho) \qquad \mathcal{V} := \Upsilon(v) \\ (\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(dst)) := \mathcal{V} \\ \Gamma(!\sigma(dst)) := \mathcal{L}\end{smallmatrix}\right] \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

get-scoped-var loads the contents of the *index* local from the scope chain skipping *skip* nodes and places it in dst, after deferred NSU. The label of the value in dst includes the join of the current context along with all the structure label of objects in the skipped nodes.

$$\text{put-scoped-var:}\ \frac{\begin{array}{c}\iota = \text{“put-scoped-var index:n skip:n value:r”} \\ \Gamma(!\sigma(value)) \neq \star \qquad \sigma' := putScopedVar(\Gamma(!\rho), \sigma, \theta, index, skip, value) \\ \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

put-scoped-var puts the contents of the *value* in the *index* local in the scope chain skipping *skip* nodes. The label of the value includes the join of the current context along with the structure label of all the objects in the skipped nodes.

$$\text{push-scope:}\ \frac{\begin{array}{c}\iota = \text{“push-scope scope:r”} \\ \sigma' := pushScope(\Gamma(!\rho), \sigma, scope) \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

push-scope converts *scope* to object and pushes it onto the top of the current scope chain. The contents of the register *scope* are replaced by the created object. The scope chain pointer label is set to the context.

$$\text{pop-scope:}\ \frac{\begin{array}{c}\iota = \text{“pop-scope”} \\ \sigma' := popScope(\Gamma(!\rho), \sigma) \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

pop-scope removes the top item from the current scope chain if the scope chain pointer label is greater than or equal to the context.

$$\text{jmp-scope:}\ \frac{\begin{array}{c}\iota = \text{“jmp-scope count:n target:n”} \\ \sigma' := jmpScope(\Gamma(!\rho), \sigma, count) \qquad \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

jmp-scope removes the top *count* items from the current scope chain if the scope chain pointer label is greater than or equal to the context. It then jumps to offset specified by *target*.

$$\text{throw:}\ \frac{\begin{array}{c}\iota = \text{“throw ex:r”} \qquad excValue := \Upsilon(!\sigma(ex)) \\ (\sigma', \iota') := throwException(\sigma, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

throw throws an exception and points to the exception handler as the next instruction to be executed, if any. The exception handler might be in the same function or in an earlier function. If it is not present, the program terminates. If it has an exception handler, it has an edge to the synthetic exit node. Apart from this, *throwException* pops the call-frames from the call-stack until it reaches the call-frame containing the exception handler. It writes the exception value to a local interpreter variable ($excValue$), which is then read by catch.

$$\text{catch:}\ \frac{\begin{array}{c}\iota = \text{“catch ex:r”} \\ \mathcal{L} := \Gamma(excValue) \sqcup \Gamma(!\rho) \qquad (\Gamma(!\sigma(ex)) \geq \Gamma(!\rho)) \Rightarrow (\mathcal{L} := \mathcal{L}) \diamond (\mathcal{L} := \star) \\ \sigma' := \sigma\left[\begin{smallmatrix}\Upsilon(!\sigma(ex)) := \Upsilon(excValue)) \\ \Gamma(!\sigma(ex)) := \mathcal{L}\end{smallmatrix}\right] \qquad excValue := empty \\ \iota' := Succ(!\sigma'.CFG, \iota) \qquad \rho' := isIPD(\iota', \rho, \sigma')\end{array}}{\theta, \iota, \sigma, \rho \ \rightarrow\ \theta, \iota', \sigma', \rho'}$$

catch catches the exception thrown by an instruction whose handler corresponds to the *catch* block. It reads the exception value from $excValue$ and writes into the register *ex*. If the label of the register is less than the context, a $\star$ is joined with the label. It then makes the $excValue$ empty and proceeds to execute the first instruction in the *catch* block.

3) Proofs and Results: The fields in a frame of the pc-stack are denoted by the following symbols: $!\rho.ipd$ represents the IPD field in the top frame of the pc-stack, $\Gamma(!\rho)$ returns the label field in the top frame of the pc-stack, and $!\rho.\mathcal{C}$ returns the call-frame field in the top frame of the pc-stack.

In the equivalence relation $\sim^{\beta}_{\ell}$, $\ell = L$. The level of the attacker (L) is omitted for clarity purposes from definitions and proofs.

Definition 10 (Partial bijection)**.** *A partial bijection β is a binary relation on heap locations satisfying the following properties: (1) if $(a, b) \in \beta$ and $(a, b') \in \beta$, then $b = b'$, and (2) if $(a, b) \in \beta$ and $(a', b) \in \beta$, then $a = a'$.*

Using partial bijections, we define equivalence of values, labeled values and objects.

Definition 11 (Value equivalence)**.** *Two values r_1 and r_2 are equivalent up to β, written $r_1 \sim^{\beta} r_2$ if either (1) $r_1 = a$, $r_2 = b$ and $(a, b) \in \beta$, or (2) $r_1 = r_1 = v$ where v is some primitive value.*

Definition 12 (Labeled value equivalence)**.** *Two labeled values $v_1 = r_1^{\ell_1}$ and $v_2 = r_2^{\ell_2}$ are equivalent up to β, written $v_1 \sim^{\beta} v_2$ if one of the following holds: (1) $\ell_1 = \star$ or $\ell_2 = \star$, or (2) $\ell_1 = \ell_2 = H$, or (3) $\ell_1 = \ell_2 = L$ and $r_1 \sim^{\beta} r_2$.*

The first clause of the above definition is standard for the permissive-upgrade check. It equates a partially leaked value to every other labeled value.

Objects are formally denoted as $N = (\{p_i \mapsto \{v_i, flags_i\}\}_{i=0}^{n}, \_\_proto\_\_ \mapsto a^{\ell_p}, \ell_s)$. Here p_is correspond to the property name, v_is are their respective values and $flags_i$ represent the *writable*, *enumerable* and *configurable* flags as described in the *PropertyDescriptor* structure in the cpp model above. As the current model does not allow modification of the $flags$, they are always set to $true$. Thus, we do not need to account for the $flags_i$ in the equivalence definition below. $\_\_proto\_\_$ represents a labelled pointer to the object's prototype.

Definition 13 (Object equivalence)**.** *For ordinary objects $N = (\{p_i \mapsto \{v_i,\ flags_i\}\}_{i=0}^{n},\ \_\_proto\_\_ \mapsto a^{\ell_p}, \ell_s)$ and $N' = (\{p'_i \mapsto \{v'_i, flags'_i\}\}_{i=0}^{m}, \_\_proto\_\_ \mapsto a'^{\ell'_p}, \ell'_s)$, we say $N \sim^{\beta} N'$ iff either $\ell_s = \ell'_s = H$ or the following hold: (1) $\ell_s = \ell'_s = L$, (2) $[p_0, \ldots, p_n] = [p'_0, \ldots, p'_m]$ (in particular, $n = m$), (3) $\forall i.\, v_i \sim^{\beta} v'_i$, and (4) $a^{\ell_p} \sim^{\beta} a^{\ell'_p}$.*

For function objects $F = (N, f, \Sigma)$ and $F' = (N', f', \Sigma')$, we say $F \sim^{\beta} F'$ iff either $N.\ell_s = N.\ell'_s = H$ or $N \sim^{\beta} N'$, $f =^{\beta} f'$ and $\Sigma \sim^{\beta} \Sigma'$.

The equality $f =^{\beta} f'$ of nodes f, f' in CFGs means that the portions of the CFGs reachable from f, f' are equal modulo renaming of operands to bytecodes under β. Equivalence $\Sigma \sim^{\beta} \Sigma'$ of scope chains is defined below. Because we do not allow $\star$ to flow into heaps, we do not need corresponding clauses in the definition of object equivalence.

Definition 14 (Heap equivalence)**.** *For two heaps θ_1, θ_2, we say that $\theta_1 \sim^{\beta} \theta_2$ iff $\forall (a, b) \in \beta.\ \theta_1(a) \sim^{\beta} \theta_2(b)$.*

Unlike objects, we allow $\star$ to permeate scope chains, so our definition of scope chain equivalence must account for it. Scope chains are denoted as Σ. A scope-chain node contains a label ℓ along with an object S (either JSActivation or JSObject) represented as (S, ℓ).

Definition 15 (Scope chain equivalence)**.** *For two scope chain nodes S, S', we say that $S \sim^{\beta} S'$ if one of the following holds: (1) $S = O$, $S' = O'$ and $O \sim^{\beta} O'$, or (2) $S = v_0 : \ldots : v_n$, $S' = v'_0 : \ldots : v'_n$ and $\forall i.\ v_i \sim^{\beta} v'_i$.*

Equivalence of two scope chains Σ, Σ' is defined by the following rules. (1) $nil \sim^{\beta} nil$ (2) $(nil \sim^{\beta} (S, \ell))$ if $\ell = H$ or $\ell = \star$ (3) $((S, \ell) \sim^{\beta} nil)$ if $\ell = H$ or $\ell = \star$ and (4) $((S, \ell) : \Sigma) \sim^{\beta} ((S', \ell') : \Sigma')$ if one of the following holds: (a) $\ell = \star$ or $\ell' = \star$, (b) $\ell = \ell' = H$, or (c) $\ell = \ell' = L$, $S \sim^{\beta} S'$ and $\Sigma \sim^{\beta} \Sigma'$.

Definition 16 (Call-frame equivalence)**.** *For two call frames μ_1, μ_2, we say $\mu_1 \sim^{\beta} \mu_2$ iff (1) $\#Registers(\mu_1) = \#Registers(\mu_2)$, (2) $\forall i.\ \mu_1.Registers[i] \sim^{\beta} \mu_2.Registers[i]$, (3) $\mu_1.CFG =^{\beta} \mu_2.CFG$, (4) $\mu_1.Scopechain \sim^{\beta} \mu_2.Scopechain$, (5) $\mu_1.\iota_r = \mu_2.\iota_r$ (6) $(\mu_1.\ell_c = \mu_2.\ell_c = H) \vee (\mu_1.\ell_c = \mu_2.\ell_c = L \wedge \mu_1.f_{callee} \sim^{\beta} \mu_2.f_{callee})$ (7) $\mu_1.argcount = \mu_2.argcount$ (8) $\mu_1.getter = \mu_2.getter$ and (9) $\mu_1.dReg =_{\beta} \mu_1.dReg$*

Note that a register is simply a labeled value in our semantics, so clause (2) above is well-defined.

Definition 17 (pc-stack equivalence)**.** *For two pc-stacks ρ_1, ρ_2, we say $\rho_1 \sim \rho_2$ iff the corresponding nodes of ρ_1 and ρ_2 having label L are equal, except for the call-frame $(\mathcal{C})$ field.*

In proofs that follow, two pc-stack nodes are equal if their respective fields are equal, except for the call-frame $(\mathcal{C})$ field.

Definition 18 (Call-stack equivalence)**.** *Given $\rho_1 \sim \rho_2$, suppose:*

1) *e_1 is the lowest H-labelled node in ρ_1*
2) *e_2 is the lowest H-labelled node in ρ_2*
3) *μ_1 is the node of σ_1 pointed to by e_1*
4) *μ_2 is the node of σ_2 pointed to by e_2*
5) *σ'_1 is prefix of σ_1 up to and including μ_1 or if $\Gamma(!\rho_1) = L$ or ρ_1 is empty, $\sigma'_1 = \sigma_1$*
6) *σ'_2 is prefix of σ_2 up to and including μ_2 or if $\Gamma(!\rho_2) = L$ or ρ_2 is empty, $\sigma'_2 = \sigma_2$*

then $\sigma_1 \sim^{\beta}_{\rho_1,\rho_2} \sigma_2$, iff (1) $|\sigma'_1| = |\sigma'_2|$, and (2) $\forall i \leq |\sigma'_1|.(\sigma'_1[i] \sim^{\beta} \sigma'_2[i])$.

Definition 19 (State equivalence)**.** *Two states $s_1 = \langle \theta_1, \iota_1, \sigma_1, \rho_1 \rangle$ and $s_2 = \langle \theta_2, \iota_2, \sigma_2, \rho_2 \rangle$ are equivalent, written as $s_1 \sim^{\beta} s_2$, iff $\iota_1 = \iota_2$, $\rho_1 \sim \rho_2$, $\theta_1 \sim^{\beta} \theta_2$, and $\sigma_1 \sim^{\beta}_{\rho_1,\rho_2} \sigma_2$.*

Lemma 4 (Confinement Lemma)**.** *Suppose $C = \langle \iota, \sigma, \rho \rangle$, $C' = \langle \iota', \sigma', \rho' \rangle$, $\langle \theta, C \rangle \rightarrow \langle \theta', C' \rangle$ and $\Gamma(!\rho) = H$, then $\rho \sim \rho'$, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ and $\theta \sim^{\beta} \theta'$ where $\beta = \{(a, a) \mid a \in \theta\}$*

Proof: As $\Gamma(!\rho) = H$, the L labelled nodes in the pc-stack will remain unchanged. Branching instructions pushing a new node would have label H due to monotonicity of pc-stack. Even if ι' is the IPD corresponding to the $!\rho.ipd$, it would only pop the H labelled node. Thus, the L labelled nodes will remain unchanged. Hence, $\rho \sim \rho'$.

We assume that the $!\rho$ is the first node labelled H in the context stack. For, other higher labelled nodes above the first node labelled H in the pc-stack, the call-frames corresponding to the nodes having L label in the pc-stack remain the same. Hence, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$.
By case analysis on the instruction type:

1) *prim*:
   a) If $\Gamma(!\sigma(dst)) \geq \Gamma(!\rho)$, then $\Gamma(!\sigma(dst)) = H$.
      By premise of prim, $\Gamma(!\sigma'(dst)) = H$. By Definition 12, $!\sigma(dst) \sim^{\beta} !\sigma'(dst)$.
   b) If $\Gamma(!\sigma(dst)) < \Gamma(!\rho)$, then $\Gamma(!\sigma'(dst))$ will contain a $\star$ and by Definition 12, $!\sigma(dst) \sim^{\beta} !\sigma'(dst)$.

   Only dst changes in the call-frame, so by Definition 16, $!\sigma \sim^{\beta} !\sigma'$. Also, other call-frames remain unchanged. By Definition 18, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$.
   $\theta = \theta'$, thus, $\theta \sim^{\beta} \theta'$.
2) *mov*: Similar to prim.
3) *jfalse*: $\sigma = \sigma'$ and $\theta = \theta'$, so, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ and $\theta \sim^{\beta} \theta'$.
4) *loop-if-less*: Similar to jfalse.
5) *typeof*: Similar to prim.
6) *instanceof*: Similar to prim.
7) *enter*: $\sigma = \sigma'$, so $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$. $\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
8) *ret*: If $!\sigma.getter = false$ then only $!\sigma$ is popped, the call-frames until $(!\rho.\mathcal{C})$ are unchanged. When $!\sigma.getter = true$, then it sets $!\sigma'(!\sigma.dReg)$ with $!\sigma(res)$. Now, let σ_1 is the prefix of σ such that $!\rho.\mathcal{C} =_{\beta} !\sigma_1$. If $!\sigma' \notin \sigma_1$ then changes in $!\sigma'$ does not effect the callframe equivalence and if $!\sigma' \in \sigma_1$ then $\Gamma(!\sigma'(!\sigma.dReg)) = \star$ (when $\Gamma(!\sigma_1(!\sigma.dReg))) = L$ or $\Gamma(!\sigma_1(!\sigma.dReg))) = \star$) and $\Gamma(!\sigma'(!\sigma.dReg))) = H$ (when $\Gamma(!\sigma_1(!\sigma.dReg))) = H$), each of the cases give $!\sigma_1(!\sigma.dReg) \sim^{\beta} !\sigma'(!\sigma.dReg)$ from Definition 12. So, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$, by Definition 18. $\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
9) *end*: The confinement lemma does not apply.
10) *call*: If it pushes on top of pc-stack, $!\rho.\mathcal{C}$ is the lowest H-labelled node in ρ'. If it joins the label with $!\rho$, the L labelled nodes remain unchanged and the $!\rho.\mathcal{C} = !\rho'.\mathcal{C}$. All the call-frames until $!\rho.\mathcal{C}$ remain unchanged. So, by Definition 18, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$.
    - If its a JS call:. $\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
    - If its a host call: For the native calls *invokeNativeMethod* ensures that the structure label the object (first argument of the call) is greater than the $\mathcal{PC}$. In case of DOM calls the DOM might get modified, so we need to check for the low-equivalence of that:
    - insertBefore: The newChild is inserted only if $\Gamma(newChild.parent) \geq H$, $\Gamma(newChild.nextSibling) \geq H$ and $\Gamma(newChild.previousSibling) \geq H$ and for the following cases:
      a) refChild is null: Insertion is allowed only when $\Gamma(lastChild) \geq H$ in the parent node. Also, for the existing lastChild $\Gamma(lastChild.nextSibling) \geq H$.
      b) refChild is the firstChild: Insertion is allowed only when $\Gamma(firstChild) \geq H$ in the parent node and $\Gamma(refChild.previousSibling) \geq H$ for the refChild.
      c) Otherwise: The insertion is allowed only if the $\Gamma(refChild.previous.nextSibling) \geq H$ in the previousSibling of the refChild and $\Gamma(refChild.previousSibling) \geq H$ in the refChild.

      By Definition 14, $\theta \sim^{\beta} \theta'$. γ will get the newChild as the value and its new label would be $\Gamma(newChild) \sqcup \Gamma(!\rho)$ (γ is internal to the interpreter and is handled in *call-put-result*)
    - replaceChild: The newChild replaces the oldChild only if $\Gamma(newChild.parent) \geq H$, $\Gamma(newChild.nextSibling) \geq H$ and $\Gamma(newChild.previousSibling) \geq H$ and if the oldChild is not null, $\Gamma(oldChild.parent) \geq H$, $\Gamma(oldChild.nextSibling) \geq H$ and $\Gamma(oldChild.previousSibling) \geq H$ and for the following cases:
      a) oldChild is both the firstChild and the lastChild of the parentNode: Replacement is allowed only when $\Gamma(firstChild) \geq H$, $\Gamma(lastChild) \geq H$ in the parentNode.
      b) oldChild is the firstChild of the parentNode: Replacement is allowed only when $\Gamma(firstChild) \geq H$ in the parentNode and $\Gamma(oldChild.next.previousSibling) \geq H$ for the oldChild's next sibling.
      c) oldChild is the lastChild of the parentNode: Replacement is allowed only when $\Gamma(lastChild) \geq H$ in the parentNode, and $\Gamma(oldChild.previous.nextSibling) \geq H$ for the oldChild's next sibling.

d) oldChild is some other sibling: Replacement is allowed only when $\Gamma(oldChild.previous.nextSibling) \geq H$ in the previous sibling of oldChild and $\Gamma(oldChild.next.previousSibling) \geq H$ in the next sibling of the oldChild.

In all cases $\theta \sim^{\beta} \theta'$ from Definiton 14. γ will get the oldChild as the value and its new label would either $\Gamma(oldChild) \sqcup \Gamma(!\rho)$ (γ is internal to the interpreter and is handled in *call-put-result*)

- removeChild: The oldChild is removed if the oldChild is not null and $\Gamma(oldChild.parent) \geq H$, $\Gamma(oldChild.nextSibling) \geq H$ and $\Gamma(oldChild.previousSibling) \geq H$, and for the following cases:
  a) oldChild is the only child of the parentNode: Removal is allowed only when $\Gamma(firstChild) \geq H$ and $\Gamma(lastChild) \geq H$ in the parent node.
  b) oldChild is the first child of the parentNode: Removal is allowed only when $\Gamma(firstChild) \geq H$ in the parentNode and $\Gamma(oldChild.next.previousSibling) \geq H$ for the oldChild's next sibling.
  c) oldChild is the last child of the parentNode: Removal is allowed only when $\Gamma(lastChild) \geq H$ in the parentNode and $\Gamma(oldChild.previous.nextSibling) \geq H$ for the oldChild's previous sibling.
  d) oldChild is some other sibling: Removal is allowed only when $\Gamma(oldChild.previous.nextSibling) \geq H$ in the previous sibling of oldChild and $\Gamma(oldChild.next.previousSibling) \geq H$ in the next sibling of the oldChild.

  In all cases $\theta \sim^{\beta} \theta'$ from Definiton 14. γ will get the oldChild as the value and its new label would either $\Gamma(oldChild) \sqcup \Gamma(!\rho)$ (γ is internal to the interpreter and is handled in *call-put-result*)
- appendChild: The newChild is appended only if the newChild is not null, and $\Gamma(newChild.parent) \geq H$, $\Gamma(newChild.nextSibling) \geq H$ and $\Gamma(newChild.previousSibling) \geq H$ and for the following cases:
  a) parentNode does not have any Child: Append operation is only allowed when in the parentNode $\Gamma(firstChild) \geq H$, $\Gamma(lastChild) \geq H$.
  b) parentNode has some Children: Append operation is only allowed when in the parentNode $\Gamma(lastChild) \geq H$ and $\Gamma(last.nextSibling) \geq H$ in the existing lastChild of the parentNode.

  In all cases $\theta \sim^{\beta} \theta'$ from Definiton 14.
- hasChildNodes: $\theta = \theta'$.
- cloneNode: A deep or a shallow cloning is performed and in either case the new nodes created have label $\geq H$. So, $\theta \sim^{\beta} \theta'$ from Definiton 14.
- normalize: Normalize on a given node succeeds only when all the adjacent textnodes and blanknodes next siblings and previous siblings have labels: $\Gamma(nextSibling) \geq H$ and $\Gamma(previousSibling) \geq H$.
- isSupported: $\theta = \theta'$.
- compareDocumentPosition: $\theta = \theta'$.
- isSameNode: $\theta = \theta'$.
- lookupPrefix: $\theta = \theta'$.
- isDefaultNamespace: $\theta = \theta'$.
- lookupNamespaceURI: $\theta = \theta'$.
- isEqualNode:$\theta = \theta'$.
- getFeature: $\theta = \theta'$.
- substringData: $\theta = \theta'$.
- appendData: Append operation succeeds only if the $\Gamma(data)$ in characterData $\geq H$ and $\Gamma(textNode) \geq H$. Hence, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- insertData: Insert operation succeeds only if the $\Gamma(data)$ in characterData $\geq H$ and $\Gamma(textNode) \geq H$. Hence, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- deleteData: Delete operation succeeds only if the $\Gamma(data)$ in characterData $\geq H$ and $\Gamma(textNode) \geq H$. Hence, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- replaceData: Replace operation succeeds only if the $\Gamma(data)$ in characterData $\geq H$ and $\Gamma(textNode) \geq H$. Hence, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- getAttribute: $\theta = \theta'$.
- setAttribute: Similar to replaceChild.
- removeAttribute: Similar to removeChild.
- getAttributeNode: $\theta = \theta'$.
- setAttributeNode: Similar to setAttribute.
- removeAttributeNode: Similar to removeAttribute.
- getElementsByTagName: Generates a live list wiht label $\geq H$. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- getAttributeNS: $\theta = \theta'$.

- setAttributeNS: Similar to setAttribute.
- removeAttributeNS: Similar to removeAttribute.
- getAttributeNodeNS: $\theta = \theta'$.
- setAttributeNodeNS: Similar to setAttribute.
- getElementsByTagNameNS: Similar to getElementsByTagName.
- hasAttribute: $\theta = \theta'$.
- hasAttributeNS: $\theta = \theta'$.
- splitText: Split operation succeeds only if the $\Gamma(data)$ in characterData $\geq H$. Along with that all checks in insertefore are also performed. Hence, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createElement: It creates an element node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createDocumentFragment: It creates a Document fragment node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createTextNode: It creates a text node with label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createComment: It creates a comment node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createCDATASection: It creates a CDATAsection node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createProcessingInstruction: It creates a ProcessingInstruction node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createAttribute:It creates an attribute node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- createEntityReference: It creates an entityreference node with a label atleast H and the different pointers also labeled at least H. So, from Definiton 14 $\theta \sim^{\beta} \theta'$.
- importNode: From appendChild, $\theta \sim^{\beta} \theta'$.
- createElementNS: Similar to createElement.
- createAttributeNS: Similar to createAttribute.
- getElementById: $\theta = \theta'$.
- adoptNode: From appendChild and removeNode, $\theta \sim^{\beta} \theta'$.
- normalizeDocument: similar to normalize.
- renameNode: This case handled by reasoning from replaceChild and removeChild.
- addEventListener: Can add a event listener only for H node. Thus, low parts of θ remain the same. Hence, $\theta \sim^{\beta} \theta'$
- removeEventListener: Can remove a event listener only for H node. Thus, low parts of θ remain the same. Hence, $\theta \sim^{\beta} \theta'$

11) *call-put-result*: Similar to prim.
12) *call-eval*: If it is a user-defined eval, it is similar to call.
In strict mode, it pushes a node on scope-chain with label H if $\Gamma(!\sigma'.\Sigma) = H$, else labels it $\star$. In non-strict mode, it does not push a node on the scope-chain. $!\sigma$ remains equivalent with corresponding call-frame in σ' by Definition 16. As other L call-frames are unchanged, by Definition 18, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$.
$\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
13) *create-arguments*: Over the initial β, by Definition 14, $\theta \sim^{\beta} \theta'$. If the argument object is created at x, then $\beta = (x, x) \cup \beta$ after the step is taken.
$\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ (Similar to prim).
14) *new-func*: Over the initial β, by Definition 14, $\theta \sim^{\beta} \theta'$. If the function object is created at x, then $\beta = (x, x) \cup \beta$ after the step is taken.
$\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ (Similar to prim).
15) *create-activation*: Over the initial β, by Definition 14, $\theta \sim^{\beta} \theta'$. If the argument objects is created at x, then $\beta = (x, x) \cup \beta$ after the step is taken.
It puts the object in dst with label H or $\star$, depending on dst value's initial label. Also, pushes a node containing the object in the scope chain with a $\star$, if $\Gamma(!\sigma.\Sigma) = L \vee \star$ or with label H, if $\Gamma(!\sigma.\Sigma) = H$ or $(!\sigma.\Sigma) = nil$. Thus, $!\sigma.\Sigma \sim^{\beta} !\sigma'.\Sigma$ by Definition 15. By Definition 16, $!\sigma \sim^{\beta} !\sigma'$. Other call-frames are unchanged, so $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ by Definition 18.

16) *construct*: Similar to call.
17) *create-this*: Similar to create-arguments.
18) *new-object*: Over the initial β, by Definition 14, $\theta \sim^{\beta} \theta'$. If the new object is created at x, then $\beta = (x, x) \cup \beta$ after the step is taken.
$\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ (Similar to prim).
19) *get-by-id*: Similar to mov when the property is a data property. If the property is an accessor property then getter is invoked and if the invocation of getter pushes an entry on top of pc-stack, $!\rho.\mathcal{C}$ remains the lowest H-labelled node in ρ'. If it joins the label with $!\rho$, the L labelled nodes remain unchanged and the $!\rho.\mathcal{C} = !\rho'.\mathcal{C}$. All the call-frames until $!\rho.\mathcal{C}$ remain unchanged. So, by Definition 18, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$. $\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
20) *put-by-id*: Sets the property of the object *base* object to the *value* with label H if the structure label of the object $\ell_s = H$. Thus, the object remains low-equivalent by Definition 13. Thus, $\theta \sim^{\beta} \theta'$ by Definition 14.
Also, $\sigma = \sigma'$, so, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$.
21) *del-by-id* : Deletes the property if structure label of object, $\ell_s = H$. Thus, the object remains low-equivalent by Definition 13. By Definition 14, $\theta \sim^{\beta} \theta'$.
$\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ (Similar to mov).
22) *getter-setter*: Sets accessor property of the object *base* object with $\Gamma(getter)$ and $\Gamma(setter)$ and label H if the structure label of the object $\ell_s = H$. Thus, the object remains low-equivalent by Definition 13. Thus, $\theta \sim^{\beta} \theta'$ by Definition 14. Also, $\sigma = \sigma'$, so, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$.
23) *get-pnames*: Similar to mov and jfalse.
24) *next-pname*: Similar to mov.
25) *resolve*: If the property exists, it is similar to mov. If it does not, it is similar to throw.
26) *resolve-skip*: Similar to resolve.
27) *resolve-global*: Similar to resolve.
28) *resolve-base*: Similar to resolve.
29) *resolve-with-base*: Similar to resolve.
30) *get-scoped-var*: Similar to mov.
31) *put-scoped-var*: Writes the value in the *index*th register in *skip*th node. If $\Gamma(!\sigma(index)) = H$, then, $\Gamma(!\sigma'(index)) = H$. Else if $\Gamma(!\sigma(index)) = L$, then, $\Gamma(!\sigma'(index)) = \star$. Other call-frames are unchanged. Thus, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ by Definition 16 and 18.
$\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
32) *push-scope*: Pushes node on scope-chain with label H if $\Gamma(!\sigma.\Sigma) = H$ or $(!\sigma.\Sigma) = nil$. Else, assigns a $\star$ as the label. Thus, $!\sigma.\Sigma \sim^{\beta} !\sigma'.\Sigma$. Registers remain unchanged. By Definition 16, $!\sigma \sim^{\beta} !\sigma'$. Other call-frames are unchanged, so by Definition 18, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$. $\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
33) *pop-scope*: Pops the node from the scope-chain if $\Gamma(!\sigma.\Sigma) = H \vee \star$. Registers remain unchanged. By Definition 16, $!\sigma \sim^{\beta} !\sigma'$. Other call-frames are unchanged, so by Definition 18, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$. $\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
34) *jmp-scope*: Similar to pop-scope.
35) *throw*: Pops the call-frames until the handler is reached, i.e., until $(!\rho.\mathcal{C})$. The property of IPD ensures that $!\sigma' = (!\rho.\mathcal{C})$. Either $!\rho'.ipd = !\rho.ipd$ or $\iota' = !\rho.ipd$. Thus, $!\rho.\mathcal{C}$ is $!\sigma'$. This call-frame and the ones below remain unchanged. Thus, $\sigma \sim^{\beta}_{\rho,\rho'} \sigma'$ by Definition 18.
$\theta = \theta'$, so $\theta \sim^{\beta} \theta'$.
36) *catch*: Similar to mov.

■

Corollary 3. *If* $\langle \theta_0, \iota_0, \sigma_0, \rho_0 \rangle \rightarrow^n \langle \theta_n, \iota_n, \sigma_n, \rho_n \rangle$ *and* $\forall (0 \leq i \leq n).\Gamma(!\rho_i) = H$, *then* $\rho_0 \sim \rho_n$, *and* $\sigma_0 \sim^{\beta}_{\rho_0,\rho_n} \sigma_n$

Proof: To prove: $\rho_0 \sim \rho_n$.
Proof by induction on n.
Basis: $\rho_0 \sim \rho_0$
IH : $\rho_0 \sim \rho_{n-1}$
From Definition 17, L labelled nodes of ρ_0 and ρ_{n-1} are equal. From Lemma 4, $\rho_{n-1} \sim \rho_n$ so, L labelled nodes of ρ_{n-1} and ρ_n are equal. Thus, L labelled nodes of ρ_0 and ρ_n are equal and by Definition 17, $\rho_0 \sim \rho_n$.

To prove: $\sigma_0 \sim^{\beta}_{\rho_0,\rho_n} \sigma_n$.
Basis: $\sigma_0 \sim^{\beta}_{\rho_0,\rho_0} \sigma_0$.

IH: $\sigma_0 \sim^{\beta}_{\rho_0,\rho_{n-1}} \sigma_{n-1}$.
From Lemma 4, $\sigma_{n-1} \sim^{\beta}_{\rho_{n-1},\rho_n} \sigma_n$. As $\forall(0 \leq i \leq n).\Gamma(\rho_i) = H$, the lowest H-labelled node is the same (pc-stack grows monotonically) in $\rho_0, \rho_{n-1}, \rho_n$. Let the call-frames pointed to by lowest H-labelled node be $\mathcal{C}_0, \mathcal{C}_{n-1}, \mathcal{C}_n$ with call-stack size until the call-frames k (from Definition 18 size of the prefix is same and by transitivity of equality it is the same for all the three cases).
$\forall \mu_0 \in \sigma_0, \mu_{n-1} \in \sigma_{n-1}, \mu_n \in \sigma_n$ until $\mathcal{C}_0, \mathcal{C}_{n-1}, \mathcal{C}_n$ respectively with sizes k, the following conditions hold:

1) $\forall(1 \leq i \leq k).((\mu_0[i].\#Registers) = (\mu_{n-1}[i].\#Registers))$ and $\forall(1 \leq i \leq k).((\mu_{n-1}[i].\#Registers) = (\mu_n[i].\#Registers))$.
   Thus, $\forall(1 \leq i \leq k).((\mu_0[i].\#Registers) = (\mu_n[i].\#Registers))$.
2) As the number of registers is the same, given by r,
   $\forall(1 \leq i \leq k).\forall r((\mu_0[i].Registers[r]) \sim^{\beta} (\mu_{n-1}[i].Registers[r]))$ and
   $\forall(1 \leq i \leq k).\forall r.((\mu_{n-1}[i].Registers[r]) \sim^{\beta} (\mu_n[i].Registers[r]))$.
   Let $v_0^{\ell_0}$, $v_{n-1}^{\ell_{n-1}}$ and $v_n^{\ell_n}$ represents the values in the registers for σ_0, σ_{n-1} and σ_n respectively. Then from Definition 12
   a) $\ell_0 = \ell_{n-1} = H$: In this case $\ell_n = H$ and $v_0^{\ell_0} \sim^{\beta} v_n^{\ell_n}$(from Lemma 4 and Definition 12).
   b) $\ell_0 = \ell_{n-1} = L$ and $v_0 = v_{n-1}$: In this case either:
      i) $\ell_n = \star$
      ii) $\ell_n = L$ and $v_{n-1} = v_n$: In this case, the value remains unchanged.
      Thus, from Definition 12 $v_0^{\ell_0} \sim^{\beta} v_n^{\ell_n}$.
   c) $\ell_0 = \star \vee \ell_{n-1} = \star$: Now the following cases arise:
      i) $\ell_0 = \star$: $v_0^{\ell_0} \sim^{\beta} v_n^{\ell_n}$.
      ii) $\ell_{n-1} = \star$: By Lemma 4 $l_n = \star$. Thus, $v_0^{\ell_0} \sim^{\beta} v_n^{\ell_n}$.
3) $\forall(1 \leq i \leq k).((\mu_0[i].CFG) = (\mu_{n-1}[i].CFG))$ and $\forall(1 \leq i \leq k).((\mu_{n-1}[i].CFG) = (\mu_n[i].CFG))$.
   Thus, $\forall(1 \leq i \leq k).((\mu_0[i].CFG) = (\mu_n[i].CFG))$.
4) $\forall(1 \leq i \leq k).((\mu_0[i].\Sigma) \sim^{\beta} (\mu_{n-1}[i].\Sigma))$ and $\forall(1 \leq i \leq k).((\mu_{n-1}[i].\Sigma) \sim^{\beta} (\mu_n[i].\Sigma))$.
   From Definition 15:
   a) If nil_0 and nil_{n-1} be the two scope chains, then due to confinement (Lemma 4) $\mu_n[i].\Sigma = nil$ or $\mu_n[i].\Sigma = (S, \ell_n)$, where $\ell_n = H$. In either case $\forall(1 \leq i \leq k).((\mu_0[i].\Sigma) \sim^{\beta} (\mu_n[i].\Sigma))$ from Definition 15.
   b) If $((S_0, \ell_0) : \Sigma_0)$, $((S_{n-1}, \ell_{n-1}) : \Sigma_{n-1})$ and $((S_n, \ell_n) : \Sigma_n)$ be the three scope-chains, then for$((S_0, \ell_0) : \Sigma_0)$ and $((S_{n-1}, \ell_{n-1}) : \Sigma_{n-1})$ one of the following holds:
      i) $\ell_0 = \star \vee \ell_{n-1} = \star$: Due to confinement (Lemma 4) and Definition 15 $\ell_n = \star$.
      ii) $\ell_0 = \ell_{n-1} = H$: Due to confinement (Lemma 4) and Definition 15 $l_n = H$.
      iii) $\ell_0 = \ell_{n-1} = L \wedge S_0 \sim^{\beta} S_{n-1} \wedge \Sigma_0 \sim^{\beta} \Sigma_{n-1}$: Due to confinement (Lemma 4) either one should hold:
         A) $\ell_n = \star$: By Definition 15.
         B) $\ell_n = L \wedge S_{n-1} \sim^{\beta} S_n \wedge \Sigma_{n-1} \sim^{\beta} \Sigma_n$: No additions to the scope chain.
      Thus, $\forall(1 \leq i \leq k).((\mu_0[i].\Sigma) \sim^{\beta} (\mu_n[i].\Sigma))$ from Definition 15.
5) $\forall(1 \leq i \leq k).((\mu_0[i].\iota_r) = (\mu_{n-1}[i].\iota_r))$ and $\forall(1 \leq i \leq k).((\mu_{n-1}[i].\iota_r) = (\mu_n[i].\iota_r))$.
   Thus, $\forall(1 \leq i \leq k).((\mu_0[i].\iota_r) = (\mu_n[i].\iota_r))$.
6) $\forall(1 \leq i \leq k).(((\mu_0[i].\ell_c) = (\mu_{n-1}[i].\ell_c) = H) \vee (((\mu_0[i].\ell_c) = (\mu_{n-1}[i].\ell_c) = L) \wedge ((\mu_0[i].f_c) = (\mu_{n-1}[i].f_c))))$ and
   $\forall(1 \leq i \leq k).(((\mu_{n-1}[i].\ell_c) = (\mu_n[i].\ell_c) = H) \vee (((\mu_{n-1}[i].\ell_c) = (\mu_n[i].\ell_c) = L) \wedge ((\mu_{n-1}[i].f_c) = (\mu_n[i].f_c))))$.
   Then either:
   - $\forall(1 \leq i \leq k).((\mu_0[i].\ell_c) = (\mu_n[i].\ell_c) = H)$ or
   - $\forall(1 \leq i \leq k).(((\mu_0[i].\ell_c) = (\mu_n[i].\ell_c) = L) \wedge ((\mu_0[i].f_c) = (\mu_n[i].f_c)))$.
7) $\forall(1 \leq i \leq k).((\mu_0[i].argcount) = (\mu_{n-1}[i].argcount))$ and $\forall(1 \leq i \leq k).\ ((\mu_{n-1}[i].argcount) = (\mu_n[i].argcount))$.
   Thus, $\forall(1 \leq i \leq k).((\mu_0[i].argcount) = (\mu_n[i].argcount))$.
8) $\forall(1 \leq i \leq k).((\mu_0[i].getter) = (\mu_{n-1}[i].getter))$ and $\forall(1 \leq i \leq k).((\mu_{n-1}[i].getter) = (\mu_n[i].getter))$.
   Thus, $\forall(1 \leq i \leq k).((\mu_0[i].getter) = (\mu_n[i].getter))$.
9) $\forall(1 \leq i \leq k).((\mu_0[i].dReg) =_{\beta} (\mu_{n-1}[i].dReg))$ and $\forall(1 \leq i \leq k).((\mu_{n-1}[i].dReg) =_{\beta} (\mu_n[i].dReg))$.
   Thus, $\forall(1 \leq i \leq k).((\mu_0[i].dReg) =_{\beta} (\mu_n[i].dReg))$.

From Definition 16 and Definition 18, $\sigma_0 \sim^{\beta}_{\rho_0,\rho_n} \sigma_n$. ■

Corollary 4. *If* $\langle \theta_0, \iota_0, \sigma_0, \rho_0 \rangle \ \rightarrow^{\star} \ \langle \theta_n, \iota_n, \sigma_n, \rho_n \rangle$ *and* $\forall(0 \leq i \leq n).\Gamma(!\rho_i) = H$, *then* $\theta_0 \sim^{\beta} \theta_n$

Proof: By induction on n.
Basis: $\theta_0 \sim^\beta \theta_0$ by Definition 14.
IH: $\theta_0 \sim^\beta \theta_{n-1}$.
From IH and Definition 14, $\forall(a,b) \in \beta.(\theta_0(a) \sim^\beta \theta_{n-1}(b)$. From Lemma 4, $\theta_{n-1} \sim^\beta \theta_n$. Thus, $\forall(b,c) \in \beta.(\theta_{n-1}(b) \sim^\beta \theta_n(c)$
As $(a,b) \in \beta$ and $(b,c) \in \beta$, we have $(a,c) \in \beta$ because β is an identity bijection. Thus, if $\forall(a,c) \in \beta.(\theta_0(a) \sim^\beta \theta_n(c)$, then $\theta_0 \sim^\beta \theta_n$. If $\theta_0(a)$ and $\theta_{n-1}(b)$ contain an ordinary object, then for their respective structure labels ℓ_s and ℓ'_s, either:

- $\ell_s = \ell'_s = H$: If $\ell'_s = H$, then $\ell''_s = H$ by Definition 13, where ℓ''_s is the structure label of the object in $\theta_n(c)$. Thus, $\theta_0(a) \sim^\beta \theta_n(c)$.
- $\ell_s = \ell'_s = L$: $[p_0, \ldots, p_n] = [p'_0, \ldots, p'_m]$ $(n = m)$, $\forall i.\, v_i \sim^\beta v'_i$, and $a^{\ell_p} \sim^\beta a^{\ell'_p}$ for respective properties in $\theta_0(a)$ and $\theta_{n-1}(b)$.
  If $\ell'_s = L$, then $\ell''_s = L$ and $[p'_0, \ldots, p'_m] = [p''_0, \ldots, p''_k]$ $(m = k)$, $\forall i.\, v'_i \sim^\beta v''_i$, and $a^{\ell'_p} \sim^\beta a^{\ell''_p}$ for respective properties in $\theta_{n-1}(b)$ and $\theta_n(c)$.
  $\ell_s = \ell''_s = L$, $[p_0, \ldots, p_n] = [p''_0, \ldots, p''_k]$ $(n = k)$. If $\forall i.\, v_i \sim^\beta v'_i$ and $\forall i.\, v'_i \sim^\beta v''_i$, then either $\ell_i = \ell'_i = \ell''_i = H$ or $\ell_i = \ell'_i = \ell''_i = L$ and $r_i = r'_i = r''_i = n$. Also, as $a^{\ell_p} \sim^\beta a^{\ell'_p}$ and $a^{\ell'_p} \sim^\beta a^{\ell''_p}$, we have $a^{\ell_p} \sim^\beta a^{\ell''_p}$. Thus, by Definition 13 $\theta_0(a) \sim^\beta \theta_n(c)$.

If $\theta_0(a)$ and $\theta_{n-1}(b)$ contain a function object, then for their respective structure labels ℓ_s and ℓ'_s, either:

- $\ell_s = \ell'_s = H$: If $\ell'_s = H$, then $\ell''_s = H$ by Definition 13, where ℓ''_s is the structure label of the function object in $\theta_n(c)$. Thus, $\theta_0(a) \sim^\beta \theta_n(c)$.
- $\ell_s = \ell'_s = L$: $\ell''_s = L$ is the structure label of the function object in $\theta_n(c)$. Thus, $N \sim^\beta N''$ from the above result for objects. The CFGs $f =^\beta f' =^\beta f''$ and the scope chains $\Sigma \sim^\beta \Sigma''$ by Corollary 3. Thus, $\theta_0(a) \sim^\beta \theta_n(c)$.

Thus, $\theta_0 \sim^\beta \theta_n$. ■

Lemma 5 (Supporting Lemma 1)**.** *Suppose* $C_1 = \langle \iota, \sigma_1, \rho_1 \rangle$, $C_2 = \langle \iota, \sigma_2, \rho_2 \rangle$,
$C'_1 = \langle \iota'_1, \sigma'_1, \rho'_1 \rangle$, $C'_2 = \langle \iota'_2, \sigma'_2, \rho'_2 \rangle$
$\langle \theta_1, C_1 \rangle \rightarrow \langle \theta'_1, C'_1 \rangle$,
$\langle \theta_2, C_2 \rangle \rightarrow \langle \theta'_2, C'_2 \rangle$,
$\rho_1 \sim \rho_2$, $\Gamma(!\rho_1) = \Gamma(!\rho_2) = L$, $\Gamma(!\rho'_1) = \Gamma(!\rho'_2)$ *and* $(\sigma_1 \sim^\beta_{\rho_1,\rho_2} \sigma_2) \wedge (\theta_1 \sim^\beta \theta_2)$
then $\rho'_1 \sim \rho'_2$, *and* $\exists \beta' : ((\beta' \supseteq \beta) \wedge (\sigma'_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma'_2) \wedge (\theta'_1 \sim^{\beta'} \theta'_2))$.

Proof: Every instruction executes *isIPD* at the end of the operation. If ι'_i is the IPD corresponding to the $!\rho_i.ipd$, then it pops the first node on the pc-stack. As $\rho_1 \sim \rho_2$ and $\Gamma(!\rho_1) = \Gamma(!\rho_2)$, ι'_i would either pop in both the runs or in none. Thus, $\rho'_1 sim \rho'_2$. For instructions that push (branch), we explain in respective instructions.
Proof by case analysis on the instruction type:

1) *prim:* No new object is created, so $\beta' = \beta$.
   As $\sigma_1 \sim^\beta_{\rho_1,\rho_2} \sigma_2$, so $!\sigma_1 \sim^\beta !\sigma_2$ and $!\sigma_1(src_i) \sim^\beta !\sigma_2(src_i)$ for $i = 1, 2$. Case analysis on the definition of $\sim^\beta$ for src_i.
   - If $(\Gamma(!\sigma_1(src_1)) = \star \vee \Gamma(!\sigma_1(src_2)) = \star \vee \Gamma(!\sigma_2(src_1)) = \star \vee \Gamma(!\sigma_2(src_2)) = \star)$, then $\Gamma(!\sigma_1(dst)) = \star \vee \Gamma(!\sigma_1(dst)) = \star$. Hence, $!\sigma_1(dst) \sim^\beta !\sigma_2(dst)$ by Definition 12.
   - If $\Gamma(!\sigma_1(src_1)) = \Gamma(!\sigma_1(src_2)) = H$ and $\Gamma(!\sigma_2(src_1)) = \Gamma(!\sigma_2(src_2)) = H$ $\Gamma(!\sigma_1(dst)) = \Gamma(!\sigma_2(dst)) = H$. So, $!\sigma_1(dst) \sim^\beta !\sigma_2(dst)$ by Definition 12.
   - If $!\sigma_1(src_1) = !\sigma_2(src_1) \wedge \Gamma(!\sigma_1(src_2)) = H \wedge \Gamma(!\sigma_2(src_2)) = H$, then $\Gamma(!\sigma_1(dst)) = \Gamma(!\sigma_2(dst)) = H$. So, $!\sigma_1(dst) \sim^\beta !\sigma_2(dst)$ by Definition 12.
     Symmetrical reasoning for $!\sigma_1(src_2) = !\sigma_2(src_2) \wedge \Gamma(!\sigma_1(src_1)) = H \wedge \Gamma(!\sigma_2(src_1)) = H$.
   - $!\sigma_1(src_1) = !\sigma_2(src_1) \wedge !\sigma_1(src_2) = !\sigma_2(src_2)$:
     $!\sigma_1(dst) = !\sigma_2(dst)$. So, $!\sigma_1(dst) \sim^\beta !\sigma_2(dst)$ by Definition 12.

   Only dst changes in the top call-frame of both the call-stacks. Thus, by Definition 16, $!\sigma'_1 \sim^\beta !\sigma'_2$. Other call-frames in σ'_1 and σ'_2 are unchanged. By Definition 18, $\sigma'_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma'_2$.
   $\theta_1 = \theta'_1$ and $\theta_2 = \theta'_2$, so, $\theta'_1 \sim^{\beta'} \theta'_2$.
2) *mov:* Similar reasoning as prim with single source.
3) *jfalse:* No new object is created, so $\beta' = \beta$.
   - $!\sigma_1(cond) = !\sigma_2(cond) \wedge \Gamma(!\sigma_1(cond)) = \Gamma(!\sigma_2(cond)) = L$: L is the label to be pushed on ρ.
   - $\Gamma(!\sigma_1(cond)) = \Gamma(!\sigma_2(cond)) = H$: H is the label to be pushed on ρ.

The IPD of ι would be the same as we have same CFG in both the cases. If the IPD is SEN, then we join the label of $!\rho_i$ with the label obtained above, which is the same in both the runs. Thus, $\Gamma(!\rho'_1) = \Gamma(!\rho'_2)$. Because $\rho_1 \sim \rho_2$, $\rho'_1 \sim \rho'_2$.

If the IPD is not SEN, then it is some other node in the same call-frame. Thus the ipd field is also the same. The $\mathcal{H}$ field is $false$ in both the cases. Thus, the pushed node is the same in both the cases and hence, $\rho'_1 \sim \rho'_2$. As, $\Gamma(!\rho'_1) = \Gamma(!\rho''_2)$, either $\iota'_1 = \iota'_2 = IPD(\iota)$ or ι'_1 and ι'_2 may or may not be equal.

$\sigma'_1 = \sigma_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma_2 = \sigma'_2$. $\theta'_1 = \theta_1 \sim^{\beta'} \theta_2 = \theta'_2$.

4) *loop-if-less:* Similar reasoning as jfalse.
5) *type-of:* Similar to mov.
6) *instance-of:* No new object is created, so $\beta' = \beta$.

   The label of the value in the dst is the label of the context joined with the label of all the prototype chain pointers traversed. As $!\sigma_1(value) \sim^{\beta} !\sigma_2(value)$, where ℓ_s and ℓ'_s are the structure labels of objects pointed to by $!\sigma_1(value)$ and $!\sigma_2(value)$ respectively, then by Definition 13:
   - If $\ell_s = \ell'_s = H$, then $\Gamma(!\sigma'_1(dst)) = H$ and $\Gamma(!\sigma'_2(dst)) = H$. So, $!\sigma'_1(dst) \sim^{\beta} !\sigma'_2(dst)$ from Definition 12.
   - If $\ell_s = \ell'_s = L$, then the objects have similar properties and prototype chains. If it is not an instance and none of traversed prototype chain and objects are H, then $\Gamma(!\sigma'_1(dst)) = \Gamma(!\sigma'_2(dst)) = L$ and $false$. Else if it is present it has $true$. So, $!\sigma'_1(dst) \sim^{\beta} !\sigma'_2(dst)$ from Definition 12. If any one of traversed prototype chain and objects are H, then $\Gamma(!\sigma'_1(dst)) = \Gamma(!\sigma'_2(dst)) = H$. So, $!\sigma'_1(dst) \sim^{\beta} !\sigma'_2(dst)$ from Definition 12.

   Only dst changes in the top call-frame of both the call-stacks. Thus, by Definition 16, $!\sigma'_1 \sim^{\beta} !\sigma'_2$. Other call-frames are unchanged and by Definition 18, $\sigma'_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma'_2$.

   $\theta_1 = \theta'_1$ and $\theta_2 = \theta'_2$, so, $\theta'_1 \sim^{\beta'} \theta'_2$.
7) *enter:* No new object is created, so $\beta' = \beta$.

   $\sigma'_1 = \sigma_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma_2 = \sigma'_2$. $\theta'_1 = \theta_1 \sim^{\beta'} \theta_2 = \theta'_2$.
8) *ret:* No new object is created, so $\beta' = \beta$.

   Since $\sigma_1 \sim^{\beta} \sigma_2$ so only two cases arise for the getter flag.
   - $!\sigma_1.getter = !\sigma_2.getter = false$: σ'_1 is same as σ_1 with $!\sigma_1$ popped. Similarly, σ'_2 is same as σ_2 with $!\sigma_2$ popped. As other call-frames are unchanged by Definition 18, $\sigma'_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma'_2$.
   - $!\sigma_1.getter = !\sigma_2.getter = true$: only resgister which changes is the $\sigma'_1(!\sigma_1.dReg$ and $\sigma'_2(!\sigma_2.dReg)$. Now, if $\Gamma(!\sigma'_1(!\sigma_1.dReg)) = \Gamma(!\sigma'_2(!\sigma_2.dReg)) = H$ then $!\sigma'_1(!\sigma_1.dReg) \sim^{\beta} !\sigma'_2(!\sigma_1.dReg)$ from defintion 12. And if $\Gamma(!\sigma'_1(!\sigma_1.dReg)) = \Gamma(!\sigma'_2(!\sigma_2.dReg)) = L$ then $!\sigma'_1(!\sigma_1.dReg)) = !\sigma_1(res) \sigma'_2(!\sigma_2.dReg) = !\sigma_2(res)$ and $!\sigma_1(res) \sim^{\beta} !\sigma_2(res)$.

   $\theta'_1 = \theta_1 \sim^{\beta'} \theta_2 = \theta'_2$.
9) *end:* No σ'_i and θ'_i.
10) *call:*

    a) JS function case: No new object is created, so $\beta' = \beta$.

       Pushes the same node on both ρs (similar to jfalse). The only difference is the $\mathcal{H}$ field. As the CFGs are same, if it has an associated exception handler, we set the $\mathcal{H}$ field to $true$ in both the runs. Else, it is $false$. Thus, $!\rho'_1 = !\rho'_2$ is the node pushed on ρ and hence, $\rho'_1 \sim \rho'_2$.

       As $!\sigma_1(func) \sim^{\beta} !\sigma_2(func)$, if:
       - $(\Gamma(!\rho'_1) = H)$: As call-frames until $!\sigma_1$ and $!\sigma_2$ remain unchanged, which correspond to the $\mathcal{C}$ field in the lowest H-labelled node and $\sigma_1 \sim^{\beta}_{\rho_1,\rho_2} \sigma_2$, by Definition 18, $\sigma'_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma'_2$.
       - $(\Gamma(!\rho'_1) = L)$: Registers created in the new call-frame contain *undefined* with label L and, as $\theta_1 \sim^{\beta} \theta_2$ so the function objects $N \sim^{\beta} N'$ implying $!\sigma'_1.CFG = !\sigma'_2.CFG$ and $!\sigma'_1.\Sigma \sim^{\beta} !\sigma'_2.\Sigma$, also return addresses are the same and the callee is the same. So $!\sigma'_1 \sim^{\beta} !\sigma'_2$. Other call-frames are unchanged so, $\sigma'_1 \sim^{\beta'}_{\rho'_1,\rho'_2} \sigma'_2$.

       $\theta'_1 = \theta_1 \sim^{\beta'} \theta_2 = \theta'_2$.

    b) DOM and host function case: $\rho'_1 \sim^{\beta} \rho'_2$ and $\sigma'_1 \sim^{\beta} \sigma'_2$ (reasoning similar to previous subcase). For pure native calls $\theta'_1 = \theta_1 \sim^{\beta} \theta_2 = \theta'_2$. For impure native calls, since $!\rho_1 = !\rho_2 = L$ we know that same native method is invoked in both the runs with low-equivalent arguments. So, $\theta'_1 \sim^{\beta} \theta'_2$. To prove $\theta'_1 \sim^{\beta} \theta'_2$ for DOM calls, we have the following cases:
       - insertBefore: $\sigma_1(parentNode) \sim^{\beta} \sigma_2(parentNode)$, $\sigma_1(refChild) \sim^{\beta} \sigma_2(refChild)$ and $\sigma_1(newChild) \sim^{\beta} \sigma_2(newChild)$ and so are their respective pointers. Thus, the pointer labels are set to $!\rho'_1 = !\rho'_2$. Hence, $\theta'_1 \sim^{\beta} \theta'_2$.

- replaceChild: $\sigma_1(parentNode) \sim^\beta \sigma_2(parentNode)$, $\sigma_1(oldChild) \sim^\beta \sigma_2(oldChild)$ and $\sigma_1(newChild) \sim^\beta \sigma_2(newChild)$ and so are their respective pointers. Thus, the pointer labels are set to $!\rho'_1 = !\rho'_2$. Hence, $\theta'_1 \sim^\beta \theta'_2$
- removeChild: $\sigma_1(parentNode) \sim^\beta \sigma_2(parentNode)$ and $\sigma_1(oldChild) \sim^\beta \sigma_2(oldChild)$, and so are their respective pointers. Thus, the pointer labels are set to $!\rho'_1 = !\rho'_2$. Hence, $\theta'_1 \sim^\beta \theta'_2$
- appendChild: $\sigma_1(parentNode) \sim^\beta \sigma_2(parentNode)$ and $\sigma_1(newChild) \sim^\beta \sigma_2(newChild)$, and so are their respective pointers. Thus, the pointer labels are set to $!\rho'_1 = !\rho'_2$. Hence, $\theta'_1 \sim^\beta \theta'_2$
- hasChildNodes: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- cloneNode: As $\sigma_1(node) \sim^\beta \sigma_2(node)$ and for all its descendant nodes, the cloned nodes will also be low-equivalent along with the pointers. Thus, $\theta'_1 \sim^\beta \theta'_2$ from Definiton 14.
- normalize: As the text nodes and the blank nodes are all low-equivalent, the normalized node, which is a union of the text and blank nodes will also be low-equivalent. Thus, $\theta'_1 \sim^\beta \theta'_2$ from Definiton 14.
- isSupported: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- compareDocumentPosition: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- isSameNode: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- lookupPrefix: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- isDefaultNamespace: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- lookupNamespaceURI: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- isEqualNode: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- getFeature: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- substringData: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- appendData: As $data_1 \sim^\beta data_2$ and $\theta_1(textNode) \sim^\beta \theta_2(textNode)$, $\theta'_1(textNode) \sim^\beta \theta'_2(textNode)$. Thus, $\theta'_1 \sim^\beta \theta'_2$ from Definiton 14.
- insertData: As $data_1 \sim^\beta data_2$ and $\theta_1(textNode) \sim^\beta \theta_2(textNode)$, $\theta'_1(textNode) \sim^\beta \theta'_2(textNode)$. Thus, $\theta'_1 \sim^\beta \theta'_2$ from Definiton 14.
- deleteData: As $data_1 \sim^\beta data_2$ and $\theta_1(textNode) \sim^\beta \theta_2(textNode)$, $\theta'_1(textNode) \sim^\beta \theta'_2(textNode)$. Thus, $\theta'_1 \sim^\beta \theta'_2$ from Definiton 14.
- replaceData: As $data_1 \sim^\beta data_2$ and $\theta_1(textNode) \sim^\beta \theta_2(textNode)$, $\theta'_1(textNode) \sim^\beta \theta'_2(textNode)$. Thus, $\theta'_1 \sim^\beta \theta'_2$ from Definiton 14.
- getAttribute: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- setAttribute: Similar to replaceChild.
- removeAttribute: Similar to removeChild.
- getAttributeNode: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- setAttributeNode: Similar to setAttribute.
- removeAttributeNode: Similar to removeAttribute.
- getElementsByTagName: Generates a live list with label $\geq H$. So, from Definiton 14 $\theta \sim^\beta \theta'$.
- getAttributeNS: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- setAttributeNS: Similar to setAttribute.
- removeAttributeNS: Similar to removeAttribute.
- getAttributeNodeNS: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- setAttributeNodeNS: Similar to setAttribute.
- getElementsByTagNameNS: Similar to getElementsByTagName.
- hasAttribute: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- hasAttributeNS: $\theta'_1 = \theta_1 \sim^\beta \theta_2 = \theta'_2$.
- splitText: Split operation succeeds only if the $\Gamma(data)$ in characterData $\geq H$. Along with that all checks in insertefore are also performed. Hence, from Definiton 14 $\theta \sim^\beta \theta'$.
- createElement: As the arguments are low-equivalent, creates a new element of the same type or with the same label H. $\theta'_1(newElement) \sim^\beta \theta'_2(newElement)$. Thus, $\theta'_1 \sim^\beta \theta'_2$.
- createDocumentFragment: As the arguments are low-equivalent, creates a new document fragment with the same label (H). $\theta'_1(newDocFragment) \sim^\beta \theta'_2(newDocFragment)$. Thus, $\theta'_1 \sim^\beta \theta'_2$.
- createTextNode: As the arguments are low-equivalent, creates a new text node with the same value or with the same label (H). $\theta'_1(newTextNode) \sim^\beta \theta'_2(newTextNode)$. Thus, $\theta'_1 \sim^\beta \theta'_2$.
- createComment: As the arguments are low-equivalent, creates a new comment with the same value or with the same label (H). $\theta'_1(newComment) \sim^\beta \theta'_2(newComment)$. Thus, $\theta'_1 \sim^\beta \theta'_2$.
- createCDATASection:As the arguments are low-equivalent, creates a new CDATA section with the same value or

with the same label (H). $\theta_1'(newCDATA) \sim^{\beta} \theta_2'(newCDATA)$. Thus, $\theta_1' \sim^{\beta} \theta_2'$.

- createProcessingInstruction: As the arguments are low-equivalent, creates a new processing instruction with the same value or with the same label (H). $\theta_1'(newPI) \sim^{\beta} \theta_2'(newPI)$. Thus, $\theta_1' \sim^{\beta} \theta_2'$.
- createAttribute: As the arguments are low-equivalent, creates a new attribute with the same value or with the same label (H). $\theta_1'(newAttr) \sim^{\beta} \theta_2'(newAttr)$. Thus, $\theta_1' \sim^{\beta} \theta_2'$.
- createEntityReference: As the arguments are low-equivalent, creates a new entity reference with the same value or with the same label (H). $\theta_1'(newER) \sim^{\beta} \theta_2'(newER)$. Thus, $\theta_1' \sim^{\beta} \theta_2'$.
- importNode: From appendChild, $\theta_1' \sim^{\beta} \theta_2'$.
- createElementNS: Similar to createElement.
- createAttributeNS: Similar to createAttribute.
- getElementById: $\theta_1' = \theta_1 \sim^{\beta} \theta_2 = \theta_2'$.
- adoptNode: From appendChild and removeNode, $\theta_1' \sim^{\beta} \theta_2'$.
- normalizeDocument: similar to normalize.
- renameNode: This case handled by reasoning from replaceChild and removeChild.
- addEventListener: The arguments to the method are equivalent, thus, the nodes on which the method is called are equivalent. The event and listener function are also equivalent. Hence, $\theta_1' \sim^{\beta} \theta_2'$.
- removeEventListener: The arguments to the method are equivalent, thus, the nodes on which the method is called are equivalent and so are the event listener lists. The listener function to be removed are also equivalent. Hence, $\theta_1' \sim^{\beta} \theta_2'$.

11) *call-put-result:* Similar to move.
12) *call-eval:* $\rho_1' \sim \rho_2'$: Similar to op-call.
In strict mode, it pushes a node on the scope-chain with label L. The pushed nodes are low-equivalent. Thus, $!\sigma_1'.\Sigma \sim^{\beta} !\sigma_2'.\Sigma$ by Definition 15. In non-strict mode, it does not push anything and is similar to call. Thus, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$ and $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
13) *create-arguments:* Let the argument object be created at x and y in θ_1 and θ_2, then $\beta' = \beta \cup (x, y)$. $\Gamma(\sigma_1'(dst)) = \Gamma(\sigma_1'(dst)) = L$ and as $!\sigma_1 \sim^{\beta} !\sigma_2$, the objects are low-equivalent. Thus, $!\sigma_1' \sim^{\beta'} !\sigma_2$ by Definition 16 and $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$ by Definition 18. Also, $\theta_1' \sim^{\beta'} \theta_2'$ by Definition 14 as the objects are low-equivalent.
14) *new-func:* Let the function object be created at x and y in θ_1 and θ_2, then $\beta' = \beta \cup (x, y)$. Function objects are low-equivalent as $!\sigma_1.\Sigma \sim^{\beta} !\sigma_2.\Sigma$ and $!\sigma_1(func) \sim^{\beta} !\sigma_2(func)$. $\sigma_1'(dst) \sim^{\beta} \sigma_2'(dst)$ by Definition 12. Thus, $!\sigma_1' \sim^{\beta'} !\sigma_2$ by Definition 16 and $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$ by Definition 18. Also, $\theta_1' \sim^{\beta'} \theta_2'$ by Definition 14 as the objects are low-equivalent.
15) *create-activation:* Similar to create-arguments.
16) *construct:* Similar to call.
17) *create-this:* Similar to create-this.
18) *new-object:* Similar to create-arguments.
19) *get-by-id:* No new object is created, so $\beta' = \beta$.
As $!\sigma_1(base) \sim^{\beta} !\sigma_2(base)$, either the objects have the same properties or are labelled H because of Definition 13. In case of data property, either $\Gamma(\sigma_1'(dst)) = \Gamma(\sigma_2'(dst)) = H$ or $\Gamma(\sigma_1'(dst)) = \Gamma(\sigma_2'(dst)) = L$ and value of *prop* is the same. So, by Definition 12 $!\sigma_1'(dst) \sim^{\beta} !\sigma_2'(dst)$.
In case of an accessor property, only dst changes in the top call-frame of both the call-stacks, and $\sigma_1'(dst)) \sim^{\beta} \sigma_2'(dst))$ since $\sigma_1 \sim^{\beta} \sigma_2$ and $\theta_1 \sim^{\beta} \theta_2$ Thus, by Definition 16, $!\sigma_1' \sim^{\beta} !\sigma_2'$. Other call-frames are unchanged and by Definition 18, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. For $\rho_1' \sim \rho_2'$, reasoning is similar to *call*. $\theta_1 = \theta_1'$ and $\theta_2 = \theta_2'$, so, $\theta_1' \sim^{\beta'} \theta_2'$.
20) *put-by-id:* No new object is created, so $\beta' = \beta$.
$\sigma_1' = \sigma_1 \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2 = \sigma_2'$.
Because $\sigma_1 \sim^{\beta}_{\rho_1,\rho_2} \sigma_2$, if *value* is labelled H, then the properties created or modified will have label H, and structure labels of the respective objects will become H. Else if *value* is labelled L, then the properties created or modified will have same value and label L. Thus, the objects remain low-equivalent by Definition 13 and hence, by Definition 14, $\theta_1' \sim^{\beta'} \theta_2'$.
21) *del-by-id:* No new object is created, so $\beta' = \beta$.
If the deleted property is H or if the structure label of the object is H , then $(\Gamma(\sigma_1'(dst)) = \Gamma(\sigma_2'(dst)) = H)$. Else if is labelled L, then $(\Gamma(\sigma_1'(dst)) = \Gamma(\sigma_2'(dst)) = L)$ and value is *true* or *false* depending on whether the property is deleted or not. $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$ by Definition 12, 16 and 18. If structure labels of the objects are L, they have same properties by Definition 13. If not, they have structure label as H. Thus, objects remain low-equivalent by Definition 13

and $\theta_1' \sim^{\beta'} \theta_2'$ by Definition 14.

22) *put-getter-setter:* Reasoning similar to put-by-id.
23) *get-pnames:* No new object is created, so $\beta' = \beta$.
As $\sigma_1 \sim^{\beta}_{\rho_1,\rho_2} \sigma_2$, $!\sigma_1(base) \sim^{\beta} !\sigma_2(base)$ and so are the objects (*obj*1 and *obj*2), $obj1 \sim^{\beta} obj2$, as $\theta_1 \sim^{\beta} \theta_2$. Thus, the structure label of the object is either H in both the runs or L and have the same properties with values (Definition 13. The IPD in both the cases is the same and so is the $\mathcal{C}$ field. The mH field is set to $false$. Thus, $\rho_1' \sim \rho_2'$.
For $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$, it is similar to mov, but done for dst, i and size.
$\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
24) *next-pname:* Similar to mov, but done for dst and base.
25) *resolve:* No new object is created, so $\beta' = \beta$.
If property is found in a L object and the scope-chain node labels are also L, then the property value is the same as $!\sigma_1 \sim^{\beta} !\sigma_2$. If it is in H object or any scope-chain node labels are H or have a $\star$, then label of the property is H or $\star$. Thus, $!\sigma_1'(dst) \sim^{\beta} !\sigma_2'(dst)$ by Definition 12. Thus, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. If property is not found in both runs, it is similar to throw. If property is not found in second run, then in the first run the property is in H context. So, the exception thrown is also H. Until, the call-frame of $!\rho_2'.ipd$, call-frames are unchanged, so $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
26) *resolve-skip:* Similar to resolve.
27) *resolve-global:* Similar to resolve.
28) *resolve-base:* Similar to resolve.
29) *resolve-with-base:* Similar to resolve.
30) *get-scoped-var:* No new object is created, so $\beta' = \beta$.
Reads *index*th register in object in *skip*th node in the scope-chain and writes into dst. As $(!\sigma_1.\Sigma \sim^{\beta} !\sigma_2.\Sigma)$, the value, if labelled L is the same, else is labelled H or $\star$. By Definition 12, $!\sigma_1'(dst) \sim^{\beta} !\sigma_2'(dst)$ and by Definition 16, $!\sigma_1' \sim^{\beta} !\sigma_2'$. Thus, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
31) *put-scoped-var:* No new object is created, so $\beta' = \beta$.
Writes into the scope chain node the same value, if "value" is labelled L. If it is labelled $\star$ in any of the runs, scope chains remain equivalent. If value is H, it checks the label of register and puts the value with label H or $\star$. Thus, $(!\sigma_1'.\Sigma \sim^{\beta} !\sigma_2'.\Sigma)$ by Definition 15 and $(!\sigma_1' \sim^{\beta} !\sigma_2')$ by Definition 16. By Definition 18, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
32) *push-scope:* No new object is created, so $\beta' = \beta$.
Pushes in the scope-chain a node containing the object in "scope" with node label L. As $(!\sigma_1(scope) \sim^{\beta} !\sigma_2(scope))$ and $(!\sigma_1.\Sigma \sim^{\beta} !\sigma_2.\Sigma)$, $(!\sigma_1'.\Sigma \sim^{\beta} !\sigma_2'.\Sigma)$ by Definition 15. Registers and other call-frames remain the same, so, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
33) *pop-scope:* No new object is created, so $\beta' = \beta$.
Pops a node from the scope chain if $\Gamma(!\sigma_1.\Sigma) = \Gamma(!\sigma_1.\Sigma) \neq (\star \vee H)$. As $(!\sigma_1.\Sigma \sim^{\beta} !\sigma_2.\Sigma)$, so $(!\sigma_1'.\Sigma \sim^{\beta} !\sigma_2'.\Sigma)$. Other registers remain the same, so, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$. $\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
34) *jmp-scope:* Similar to pop-scope.
35) *throw:* No new object is created, so $\beta' = \beta$.
The property of IPD ensures that $!\sigma_1' = (!\rho_1.\mathcal{C})$ and $!\sigma_2' = (!\rho_2.\mathcal{C})$. The $!\sigma_1'$ and $!\sigma_2'$ and the ones below them remain unchanged. Thus, $\sigma_1' \sim^{\beta'}_{\rho_1',\rho_2'} \sigma_2'$ by Definition 18.
$\theta_1' = \theta_1 \sim^{\beta'} \theta_2 = \theta_2'$.
36) *catch:* Similar to mov.

■

Lemma 6 (Supporting Lemma 2)**.** *Suppose*
$C_0' = \langle \iota_0, \sigma_0', \rho_0' \rangle$, $C_0'' = \langle \iota_0, \sigma_0'', \rho_0'' \rangle$,
$C_1' = \langle \iota_1', \sigma_1', \rho_1' \rangle$, $C_2' = \langle \iota_2', \sigma_2', \rho_2' \rangle$,
$C_n' = \langle \iota_n', \sigma_n', \rho_n' \rangle$, $C_m'' = \langle \iota_m'', \sigma_m'', \rho_m'' \rangle$ *and*

1) $\langle \theta_0', C_0' \rangle \rightarrow \langle \theta_1', C_1' \rangle \rightarrow^{n-1} \langle \theta_n', C_n' \rangle$,
2) $\langle \theta_0'', C_0'' \rangle \rightarrow \langle \theta_1'', C_1'' \rangle \rightarrow^{m-1} \langle \theta_m'', C_m'' \rangle$,
3) $(\rho_0' \sim \rho_0'')$, $(\sigma_0' \sim^{\beta}_{\rho_0',\rho_0''} \sigma_0'')$, $(\theta_0' \sim^{\beta} \theta_0'')$,
4) $(\Gamma(!\rho_0') = \Gamma(!\rho_0'') = L)$, $(\Gamma(!\rho_n') = \Gamma(!\rho_m'') = L)$,
5) $\forall(0 < i < n).(\Gamma(!\rho_i') = H) \wedge \forall(0 < j < m).(\Gamma(!\rho_j'') = H)$,

then
$(\iota'_n = \iota''_m)$, $(\rho'_n \sim \rho''_m)$, $(\sigma'_n \sim^{\beta}_{\rho'_n,\rho''_m} \sigma''_m)$, *and* $(\theta'_n \sim^{\beta} \theta''_m)$.

Proof: Starting with the same instruction and high context in both the runs, we might get two different instructions, ι'_1 and ι''_1. This is only possible if ι was some branching instruction in the first place and this divergence happened in a high context. Now,

1) To prove $\iota'_n = \iota''_m$:
From the property of the IPDs we know that if ι_0 pushes a H node on top of pc-stack which was originally L, $IPD(\iota_0)$ pops that node. Since we start from the same instrucion ι_0, $\iota'_n = \iota''_m = IPD(\iota)$, where $\Gamma(!\rho) = L$.
2) To prove $\rho'_n \sim \rho''_m$:
   - $n > 1$ and $m > 1$: $\Gamma(!\rho'_1) = \Gamma(!\rho''_1)$, because ι_0 pushes equal nodes and ι'_1, ι''_1 are not the IPDs. As $\rho'_0 \sim \rho''_0$ and $\Gamma(!\rho'_1) = \Gamma(!\rho''_1)$, from Lemma 5 we get $\rho'_1 \sim \rho''_1$ and $!\rho'_1.ipd = !\rho''_1.ipd = IPD(\iota_0)$, if $\iota'_1 \neq IPD(\iota_0)$ and $\iota''_1 \neq IPD(\iota_0)$. As $\iota'_n = \iota''_m = IPD(\iota_0)$, it pops the $!\rho'_1$ and $!\rho''_1$, which correspond to ρ'_n and ρ''_m in the nth and mth step. (IPD is the point where we pop the final H node on the pc-stack.) Because $\rho'_1 \sim \rho''_1$ and from Corollary 3, $\rho'_n \sim \rho''_m$.
   - $n = 1$ and $m > 1$: If $\Gamma(!\rho'_1) \neq \Gamma(!\rho''_1)$ and $\iota'_1 = IPD(\iota_0)$, then $\Gamma(!\rho'_1) = L$. It pops the node pushed by ι_0, i.e., $\Gamma(!\rho'_n) = L$. In the other run as $\Gamma(!\rho''_1) = H$ and $\Gamma(!\rho''_m) = L$, by the property of IPD $\iota''_m = IPD(\iota_0)$, which would pop from the pc-stack $!\rho''_1$, the first frame labelled H on the pc-stack. Thus, $\rho'_n \sim \rho''_m$.
   - $n > 1$ and $m = 1$: Symmetric case of the above.
3) To prove $\sigma'_n \sim^{\beta}_{\rho'_n,\rho''_m} \sigma''_m$:
   a) $n > 1$ and $m > 1$: From Lemma 5 we get $\sigma'_1 \sim^{\beta}_{\rho'_1,\rho''_1} \sigma''_1$. From Corollary 3 we get $\sigma'_1 \sim^{\beta}_{\rho'_1,\rho'_{n-1}} \sigma'_{n-1}$. And from Lemma 4 we have $\sigma'_{n-1} \sim^{\beta}_{\rho'_{n-1},\rho'_n} \sigma'_n$. As, $\iota'_n = \iota''_m = IPD(\iota_0)$, we compare all call-frames of σ'_n and σ''_m. As the IPD of an instruction can lie only in the same call-frame, comparison for all call-frames in σ'_0 and σ''_0 suffice.
   $\sigma'_0 \sim^{\beta}_{\rho'_0,\rho''_0} \sigma''_0 \Rightarrow \forall i.((\mu_i \in \sigma'_0 \wedge \nu_i \in \sigma''_0), (\mu_i \sim^{\beta} \nu_i) \wedge \forall (r \in \mu_i, \nu_i).(\mu_i(r) = v_1 \wedge \nu_i(r) = v_2,\ v_1 \sim^{\beta} v_2))$.
   Let v_1 and v_2 be represented by v_n and v_m in σ'_n and σ''_m, respectively. The call-frames in σ'_n and σ''_m are represented by μ_n and ν_m, respectively.
      - We do case analysis on the different cases of Definitions 12 for v_1 and v_2, to show $v_n \sim^{\beta} v_m$. As $(\forall (1 \leq i < n).(\Gamma(!\rho'_i) = H)\ \wedge\ \forall (1 \leq j < m).(\Gamma(!\rho''_j) = H))$:
        – If $v_1 = v_2 \wedge \Gamma(v_1) = \Gamma(v_2) = L$, then either $v_n = v_m$ or $((\star = \Gamma(v_n)) \vee (\star = \Gamma(v_m)))$. By Definition 12(1), $v_n \sim^{\beta} v_m$.
        – If $\Gamma(v_1) = \Gamma(v_2) = H$, then $\Gamma(v_n) = \Gamma(v_m) = H$. By Definition 12(2), $v_n \sim^{\beta} v_m$.
        – If $\star = \Gamma(v_1)$, then $\star = \Gamma(v_n)$ and if $\star = \Gamma(v_2)$, then $\star = \Gamma(v_m)$. By Definition 12(1), $v_n \sim^{\beta} v_m$.
      - Lets S_1 and S_2 be the scopechains in σ'_0 and σ''_0. And S_n and S_m represent the scopechains in σ'_n and σ''_m, i.e., S_n and S_m are the respective scope-chains in the nth and mth step of the two runs and ℓ_n and ℓ_m are their node labels. For scope chain pointers the following cases arise:
        i) $S_1 = S_2 = nil$: In this case S_n and S_m either remain nil or its head will have a H label, because of the rules of the instructions that modify the scope-chain.
        ii) $S_1 = (s_1, \ell_1) : \Sigma_1$ and $S_2 = (s_2, \ell_2) : \Sigma_2$:
            A) $\ell_1 = \star \vee \ell_2 = \star$: In this case ℓ_n and ℓ_m will be $\star$ too.
            B) $\ell_1 = \ell_2 = H$: In this case ℓ_n and ℓ_m will be H too.
            C) $\ell_1 = \ell_2 = L \wedge \Sigma_1 \sim^{\beta} \Sigma_2$: In this case $\ell_n = \star$ and $\ell_m = \star$ or scopechains remain unchanged.
   b) $n = 1$ and $m > 1$:
   In case of *jfalse* and *loop-if-less*, $\sigma'_0 = \sigma'_1$ and $\sigma''_0 = \sigma''_1$. And in case of *get-pnames*, if $n = 1$ and $m \neq 1$, $\Upsilon(!\sigma'_0(base)) = undefined$ and $\Upsilon(!\sigma''_0(base)) \neq undefined$. Because $\sigma'_0 \sim^{\beta} \sigma''_0$, $!\sigma'_0(base) \sim^{\beta} !\sigma''_0(base)$. Hence, $\Gamma(!\sigma'_0(base)) = \Gamma(!\sigma''_0(base)) = H$. Thus, $\Gamma(!\sigma'_1(dst)) = \Gamma(!\sigma'_1(i)) = \Gamma(!\sigma'_1(size)) = H$ and similarly, $\Gamma(!\sigma''_1(dst)) = \Gamma(!\sigma''_1(i)) = \Gamma(!\sigma''_1(size)) = H$. Other registers remain unchanged and so do the other call-frames. Thus, $\sigma'_1 \sim^{\beta}_{\rho'_1,\rho''_1} \sigma''_1$. From the case (a) above, we know that if $\sigma'_1 \sim^{\beta}_{\rho'_1,\rho''_1} \sigma''_1$, then $\sigma'_n \sim^{\beta}_{\rho'_n,\rho''_m} \sigma''_m$.
   c) $n > 1$ and $m = 1$: Symmetric case of the above.
4) To prove $\theta'_n \sim^{\beta} \theta''_m$:
   a) $n > 1$ and $m > 1$: From Lemma 5 we get $\theta'_1 \sim^{\beta} \theta''_1$. From Corollary 4 we get $\theta'_1 \sim^{\beta} \theta'_{n-1}$. And from Lemma 4 we have $\theta'_{n-1} \sim^{\beta} \theta'_n$. Assume O_1 is an object at x in θ'_1 and O_2 is an object at y in θ''_1, such that $(x, y) \in \beta$ and

O_n and O_m are the respective objects in the nth and mth step of the two runs. We do case analysis on the different cases of Definitions 13 for O_1 and O_2, to show $O_n \sim^\beta O_m$.

- If $\Gamma(O_1) = \Gamma(O_2) = H$, then $\Gamma(O_n) = \Gamma(O_m) = H$. By Definition 13, $O_n \sim^\beta O_m$.
- If $O_1 = O_2 \wedge \Gamma(O_1) = \Gamma(O_2) = L$, then $O_n = O_m \wedge \Gamma(O_n) = \Gamma(O_m) = L$. By Definition 13, $O_n \sim^\beta O_m$.

Similarly, for function objects, the structure labels would remain H if they were originally H or will remain L with the same CFGs and scope-chains.

b) $n = 1$ and $m > 1$: In case of *jfalse*, *loop-if-less*, *get-pnames*, $\theta'_0 = \theta'_1$ and $\theta''_0 = \theta''_1$. Thus, $\theta'_1 \sim^\beta \theta''_1$. From the case (a) above, we know that if $\theta'_1 \sim^\beta \theta''_1$, then $\theta'_n \sim^\beta \theta''_m$.

c) $n > 1$ and $m = 1$: Symmetric case of the above.

■

Definition 20 (Trace)**.** *A trace is defined as a sequence of configurations or states resulting from a program evaluation, i.e., for a program evaluation $\mathcal{P} = s_1 \to s_2 \to \ldots \to s_n$ where $s_i = \langle \theta_i, \iota_i, \sigma_i, \rho_i \rangle$, the corresponding trace is given as $\mathcal{T}(\mathcal{P}) := s_1 :: s_2 :: \ldots :: s_n$.*

Definition 21 (Epoch-trace)**.** *An epoch-trace ($\mathcal{E}$) over a trace $\mathcal{T} = s_1 :: s_2 :: \ldots :: s_n$ where $s_i = \langle \theta_i, \iota_i, \sigma_i, \rho_i \rangle$ is defined inductively as:*

$$\mathcal{E}(nil) := \ nil$$

$$\mathcal{E}(s_i :: \mathcal{T}) := \begin{cases} s_i :: \mathcal{E}(\mathcal{T}) & \text{if} \quad \Gamma(!\rho_i) = L, \\ \mathcal{E}(\mathcal{T}) & \text{else if} \quad \Gamma(!\rho_i) = H. \end{cases}$$

Theorem 5 (Termination-Insensitive Non-interference)**.** *Suppose $\mathcal{P}$ and $\mathcal{P}'$ are two program evaluations.*

Then for their respective epoch-traces given by:
$\mathcal{E}(\mathcal{T}(\mathcal{P})) = s_1 :: s_2 :: \ldots :: s_n$,
$\mathcal{E}(\mathcal{T}(\mathcal{P}')) = s'_1 :: s'_2 :: \ldots :: s'_m$,
if $s_1 \sim^\beta s'_1$ and $n \leq m$,
then
$\exists \beta_n \supseteq \beta : \ s_n \sim^{\beta_n} s'_n$

Proof: Proof proceeds by induction on n.
Basis: $s_1 \sim^\beta s'_1$, by assumption.
IH: $s_k \sim^{\beta_k} s'_k$ where $\beta_k \supseteq \beta$.
To prove: $\exists \beta_{k+1} \supseteq \beta : \ s_{k+1} \sim^{\beta_{k+1}} s'_{k+1}$.
Let $s_k \to_i s_{k+1}$ and $s_k \to_{i'} s'_{k+1}$, then:

- $i = i' = 1$: From Lemma 5, $s_{k+1} \sim^{\beta_{k+1}} s'_{k+1}$ where $\beta_{k+1} \supseteq \beta$.
- $i > 1$ or $i' > 1$: From Lemma 6, $s_{k+1} \sim^{\beta_{k+1}} s'_{k+1}$ where $\beta_{k+1} = \beta$.

■

Corollary 6. *Suppose:*
1) $\langle \theta_1, \iota_1, \sigma_1, \rho_1 \rangle \sim^\beta \langle \theta_2, \iota_2, \sigma_2, \rho_2 \rangle$
2) $\langle \theta_1, \iota_1, \sigma_1, \rho_1 \rangle \to^* \langle \theta'_1, \texttt{end}, [], [] \rangle$
3) $\langle \theta_2, \iota_2, \sigma_2, \rho_2 \rangle \to^* \langle \theta'_2, \texttt{end}, [], [] \rangle$
Then, $\exists \beta' \supseteq \beta$ such that $\theta'_1 \sim^{\beta'} \theta'_2$.

Proof: σ_1, σ_2 and ρ_1, ρ_2 are empty at the end of $*$ steps. From the semantics, we know that in L context both runs would push and pop the same number of nodes. Thus, both take same number of steps in L context. Let k be the number of states in L context. Then in Theorem 5, $n = m = k$. Thus, $s_k \sim^{\beta_k} s'_k$, where $s_k = \langle \theta'_1, end, [], [] \rangle$ and $s'_k = \langle \theta'_2, end, [], [] \rangle$. By Definition 19, $\theta'_1 \sim^{\beta'} \theta'_2$, where $\beta' = \beta_k$. ■