

# Introduction to Multiprocessor Real-Time Systems

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## Three Kinds of Multiprocessors

	Proc. 1	Proc. 2	Proc. 3
Identical	2 GHz FPU	2 GHz FPU	2 GHz FPU
Uniform Heterogeneous	2 GHz FPU	1 GHz FPU	500 MHz FPU
Unrelated Heterogeneous	1 GHz FPU	3 GHz large cache	500 MHz I/O coproc.

### identical:

- all processors have equal **speed** and **capabilities**

### uniform heterogeneous (or homogenous):

- all processors have equal **capabilities**
- but **different speeds**

### unrelated heterogeneous:

- no regular relation assumed
- tasks may not be able to execute on all processors

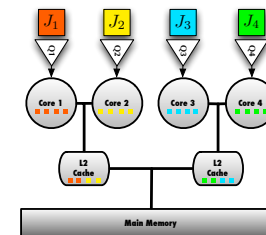
We consider only identical multiprocessors in this class.

## What makes multiprocessor scheduling hard?

*“Few of the results obtained for a single processor generalize directly to the multiple processor case; bringing in additional processors adds a new dimension to the scheduling problem. The simple fact that **a task can use only one processor** even when several processors are free at the same time adds a surprising amount of difficulty to the scheduling of multiple processors.” [emphasis added]*

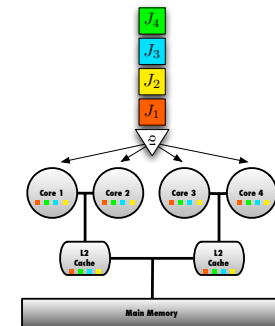
LIU, C. L. (1969). Scheduling algorithms for multiprocessors in a hard real-time environment. In JPL Space Programs Summary, vol. 37-60. JPL, Pasadena, CA, 28–31.

## Scheduling Approaches



### Partitioned Scheduling

- task **statically** assigned to cores
- One ready queue **per core**
- uniprocessor scheduler on each core



### Global Scheduling

- jobs **migrate** freely
- All cores serve **shared** ready queue
- requires new schedulability analysis

# Global Scheduling – Dhall Effect

## Uniprocessor Utilization Bounds

- EDF = 1
- Rate-Monotonic (RM) =  $\ln 2$

## Question: What are the utilization bounds on a multiprocessor?

- Notation:  $m$  is the number of processors
- Intuition: would like to **fully utilize** all processors!

## Guesses?

- Global EDF = ?
- Global RM = ?

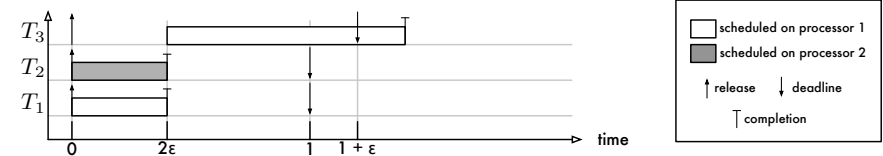
Dhall, S. and Liu, C. (1978). On a real-time scheduling problem. Operations Research, 26(1):127–140.

# Dhall Effect – Illustration

## A Difficult Task Set

- $m + 1$  tasks
- First  $m$  tasks – ( $T_i$  for  $1 \leq i \leq m$ ):
  - Period = 1
  - WCET:  $2\varepsilon$
- Last task  $T_{m+1}$ 
  - Period =  $1 + \varepsilon$
  - WCET = 1

Total utilization?



# Dhall Effect – Implications

## Utilization Bounds

- For  $\varepsilon \rightarrow 0$ , the **utilization bound approaches 1**.
- Adding processors makes no difference!

## Global vs. Partitioned Scheduling

- Partitioned scheduling is easier to implement.
- Dhall Effect shows limitation of global EDF and RM scheduling.
- Researchers lost interest in global scheduling for ~25 years.

## Since late 1990ies...

- It's a limitation of EDF and RM, not global scheduling in general.
- Much recent work on global scheduling.

# Partitioned Scheduling

## Reduction to $m$ uniprocessor problems

- Assign each task **statically** to one processor
- Use uniprocessor scheduler on each core
  - Either fixed-priority (**P-FP**) scheduling or EDF (**P-EDF**)

## Find task mapping such that

- No processor is **over-utilized**
- Each partition is **schedulable**
  - trivial for implicit deadlines & EDF

## Connection to Bin Packing

### Bin packing decision problem

Given a number of bins  $B$ , a bin capacity  $V$ , and a set of  $n$  items  $x_1, \dots, x_n$  with sizes  $a_1, \dots, a_n$ , does there exist a packing of  $x_1, \dots, x_n$  that fits into  $B$  bins?

### Bin packing optimization problem

Given a bin capacity  $V$  and a set of  $n$  items  $x_1, \dots, x_n$  with sizes  $a_1, \dots, a_n$ , assign each item to a bin such that the number of bins is minimized.

## Bin-Packing Reduction

### Bin packing decision problem

Given a number of bins  $B$ , a bin capacity  $V$ , and a set of  $n$  items  $x_1, \dots, x_n$  with sizes  $a_1, \dots, a_n$ , does there exist a packing of  $x_1, \dots, x_n$  that fits into  $B$  bins?

#### 1) Normalize sizes $a_1, \dots, a_n$ and capacity $V$

- assume unit-speed processors

#### 2) Create an implicit-deadline sporadic task $T_i$ for each item $x_i$

- with utilization  $u_i = a_i / V$
- Pick period arbitrarily, scale WCET appropriately

#### 3) Is the resulting task set feasible under P-EDF on $B$ processors?

- Hence, finding a valid partitioning is NP-hard.

## Upper Utilization Bound

**Theorem:** there exist task sets with utilizations arbitrarily close to  $(m+1)/2$  that cannot be partitioned.

Andersson, B., Baruah, S., and Jonsson, J. (2001). Static-priority scheduling on multiprocessors. In *Proceedings of the 22nd IEEE Real-Time Systems Symposium*, pages 193–202.

### A difficult-to-partition task set

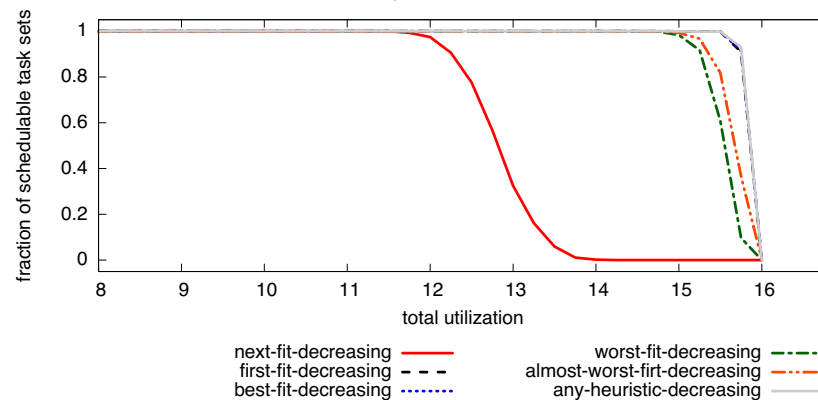
- $m + 1$  tasks
- For each  $T_i$  for  $1 \leq i \leq m + 1$ :
  - Period = 2
  - WCET:  $1 + \epsilon$
  - Utilization:  $(1 + \epsilon) / 2$

### Partitioning not possible

- Any two tasks together over-utilize a single processor by  $\epsilon!$
- Total utilization =  $(m + 1) \cdot (1 + \epsilon) / 2$

## Partitioning in Practice (I)

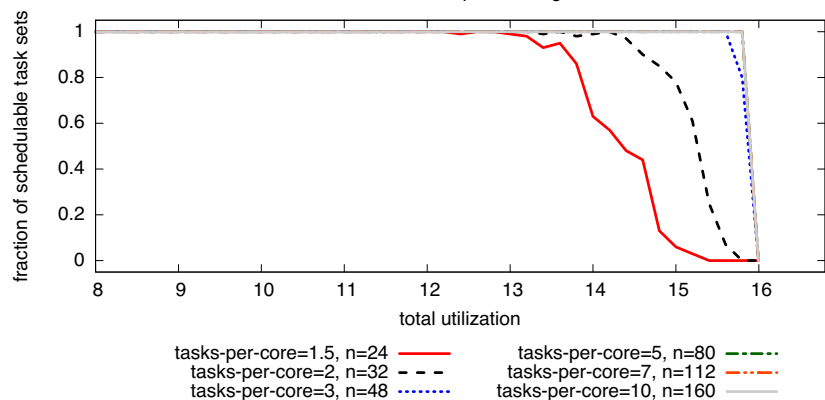
binpacking heuristics comparison (P-EDF), using Emberson et al. (2010) tasks for  $m=16$ , periods=logunif, tasks-per-core=3, and tasks=48



Bottom line: heuristics work well most of the time (for independent tasks).

## Partitioning in Practice (II)

difficulty of binpacking (P-EDF), using Emberson et al. (2010) tasks with  $m=16$ , and periods=logunif



Bottom line: larger number of tasks  $\rightarrow$  easier to partition.

## Improving Upon Partitioning

### Worst-Case Loss

- Partitioning may cause almost up to **50% utilization loss!**
- For **pathological task sets**, the system is half-idle!
- It gets much more difficult for non-independent task sets
  - Locks, precedence, etc.

### Can't we do better?

- Can we achieve a utilization bound of  $m$ ?
- Avoid **offline** assignment phase?
- Global scheduling...

## Global Scheduling

### General Approach

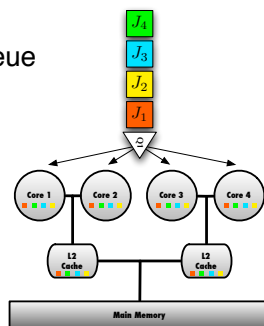
- At **each point in time**, assign **each job** a priority
- At any point in time, schedule the  $m$  highest-priority jobs

### Implementation

- Conceptually a globally shared ready queue
- Actual implementation can differ
- efficient & correct: ongoing research**

### Challenges

- migrations require coordination
- cache affinity
- lock contention
- e.g., see Linux



## Classification of Scheduling Policies

### Task-Level Fixed-Priority (FP) Scheduler (**static** priorities)

- Each **task** is assigned a fixed priority
- All jobs (of a task) have the same priority
- Example: Rate-Monotonic Scheduling

### Job-Level Fixed-Priority (JLFP) Scheduler (**dynamic** priorities)

- The priority of each task **changes over time**.
- The priority of a job does **not** change.
- Example: EDF

### Job-Level Dynamic-Priority (JLDP) Scheduler

- No restrictions.
- The priority of each job changes over time.
- Priorities are a function of **time**, **job identity**, and **system state**.

# Unknown Critical Instant

## Critical Instant

- Job release time such that **response time is maximized**.
- **Exists** unless system is over-loaded.

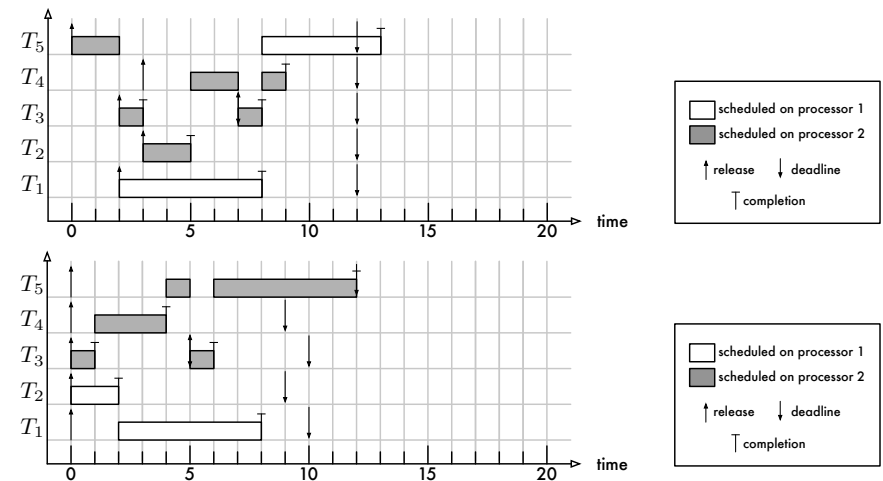
## Uniprocessor

- Liu & Layland: synchronous release sequence yields worst-case response-times
  - › synchronous: all tasks release a job at time 0
  - › *assuming constrained deadlines and no deadline misses*

## Multiprocessors

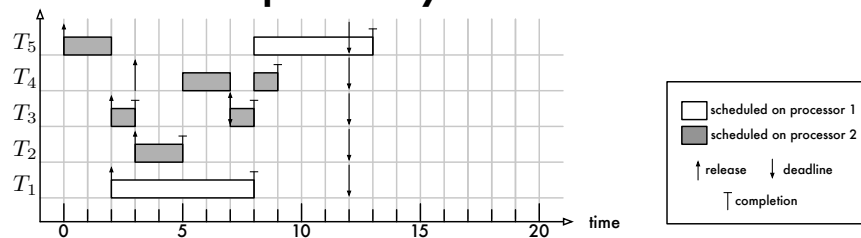
- **No general critical instant is known!**
- It is **not** necessarily the synchronous release sequence.
- A G-EDF example...

# Unknown Critical Instant



*The synchronous release sequence is not always the worst case!*

# Non-Optimality of Global EDF



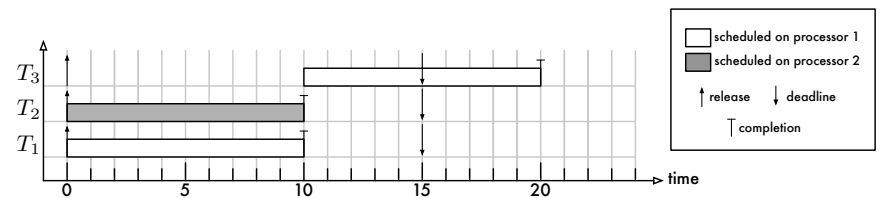
## Uniprocessor

- EDF is optimal

## Multiprocessor

- G-EDF is not optimal (w.r.t. meeting deadlines)
- Key problem: **sequentiality** of tasks
  - › Two processors available for  $T_5$ , but it can only use one.

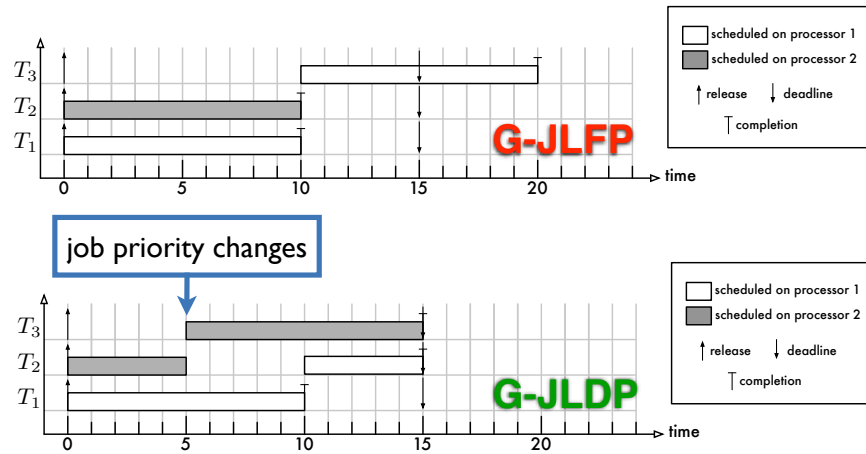
# Non-Optimality of G-JLFP Scheduling



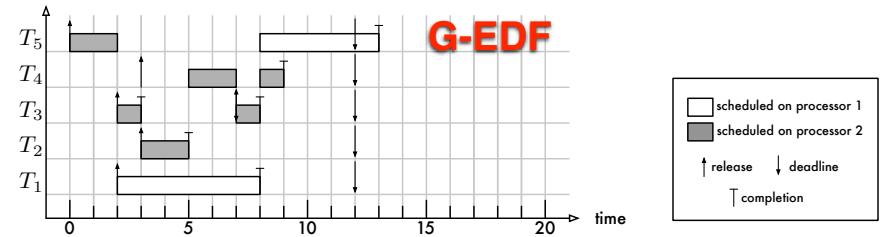
## Any Job-Level Fixed-Priority Scheduling Policy is not optimal

- Example: two processors, three tasks
  - › Period 15, WCET = 10
  - › synchronous release at time 0
- One of the three jobs is **scheduled last** under **any JLFP** policy
  - › Deadline miss inevitable!

# Global JLDP Example



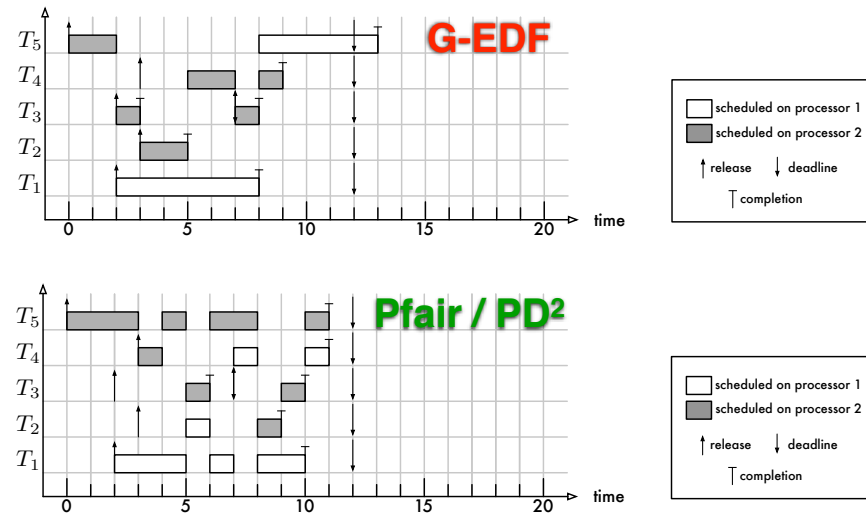
# Optimal Multiprocessor Scheduling



## G-EDF is a JLFP Policy

- Can (pseudo-)deadlines be used to schedule correctly?
- **Yes**, but deadlines alone are not enough.
  - Need to break jobs into “smaller pieces”.
  - Need appropriate **tie-breaking rules**.
- PD<sup>2</sup>

# Optimal Multiprocessor Scheduling



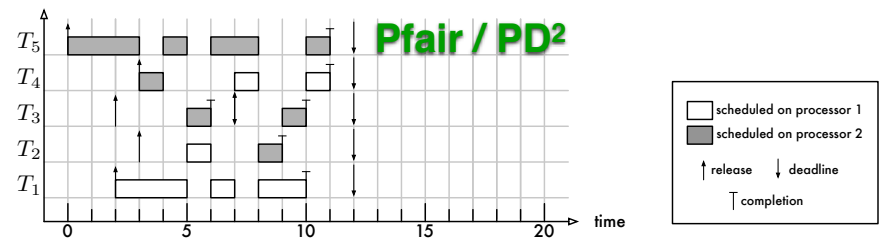
# Optimal Multiprocessor Scheduling

## Pfair

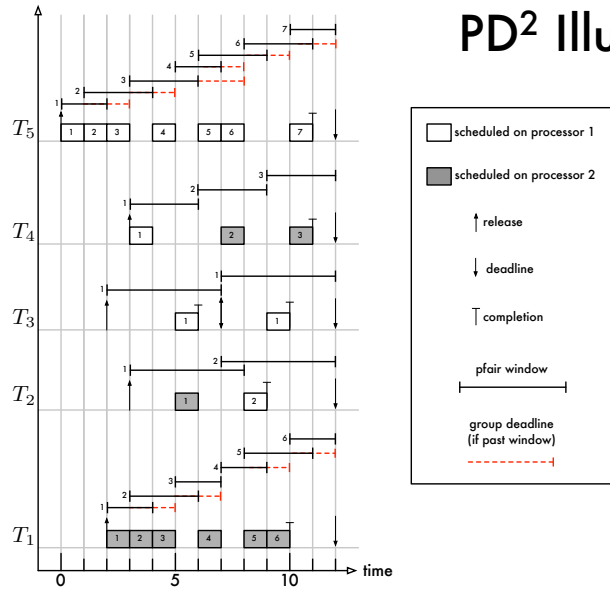
- Notion of “fair share of processor”
- If a schedule is **pfair**, then no **implicit** deadline will be missed.

## PD<sup>2</sup>

- Constructs a **pfair** schedule.
- Splits jobs into **unit-sized subtasks**
  - Each subtask has its own **deadline**
- Uses two deadline tie-breaking rules



# PD<sup>2</sup> Illustration



# Optimal Online Scheduling of Sporadic Tasks with Arbitrary Deadlines

Is it possible to extend **Pfair/PD<sup>2</sup>** to support **arbitrary deadlines**?

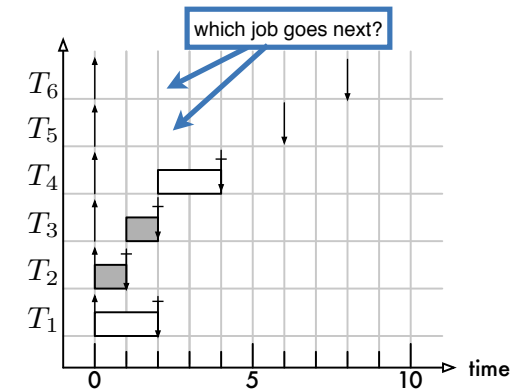
# Optimal Online Scheduling of Sporadic Tasks with Arbitrary Deadlines

**Theorem:** there does not exist an **online** scheduler that **optimally** schedules sporadic tasks with constrained deadlines.

Fisher, Goossens, Baruah (2010), Optimal online multiprocessor scheduling of sporadic real-time tasks is impossible. *Real-Time Systems*, volume 45, pp 26-71.

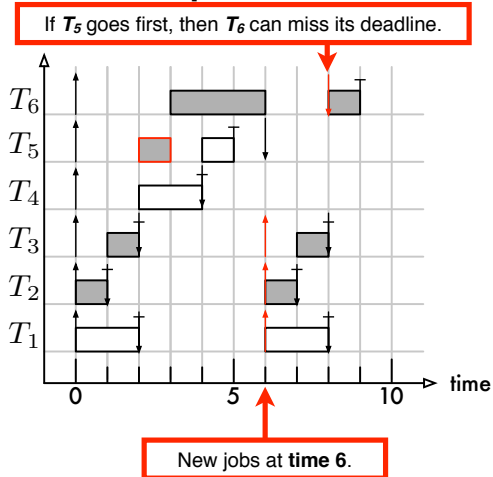
# Non-Existence of Optimal Online Schedulers for General Sporadic Tasks

Task	WCET	Deadline	Period
T <sub>1</sub>	2	2	5
T <sub>2</sub>	1	1	5
T <sub>3</sub>	1	2	6
T <sub>4</sub>	2	4	100
T <sub>5</sub>	2	6	100
T <sub>6</sub>	4	8	100



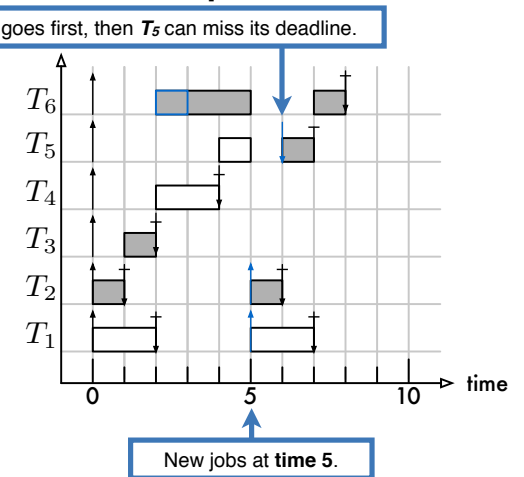
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$T_3$	1	2	6
$T_4$	2	4	100
$T_5$	2	6	100
$T_6$	4	8	100



# Non-Existence of Optimal Online Schedulers for General Sporadic Tasks

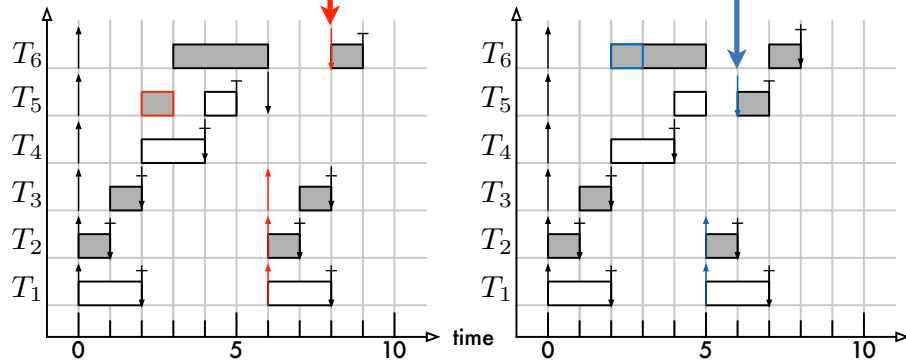
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$T_2$	1	1	5
$T_3$	1	2	6
$T_4$	2	4	100
$T_5$	2	6	100
$T_6$	4	8	100



# Non-Existence of Optimal Online Schedulers for General Sporadic Tasks

If  $T_5$  goes first, then  $T_6$  can miss its deadline.

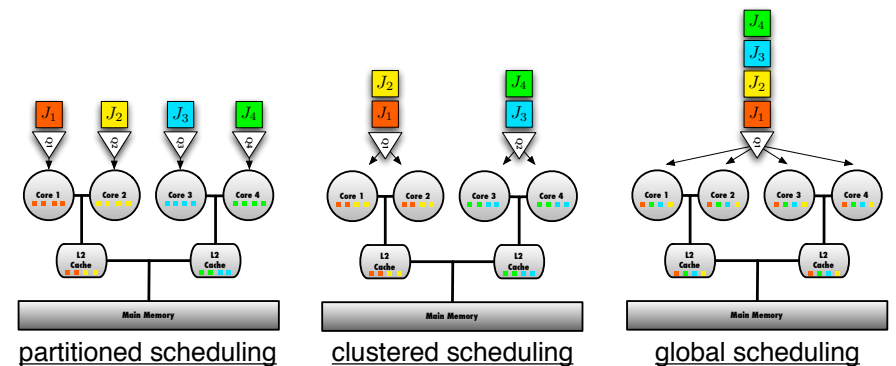
If  $T_6$  goes first, then  $T_5$  can miss its deadline.



The task set is **feasible**, but correct decision requires **knowledge of future arrivals!**

# Clustered Scheduling

A hybrid / generalization of global and partitioned scheduling.

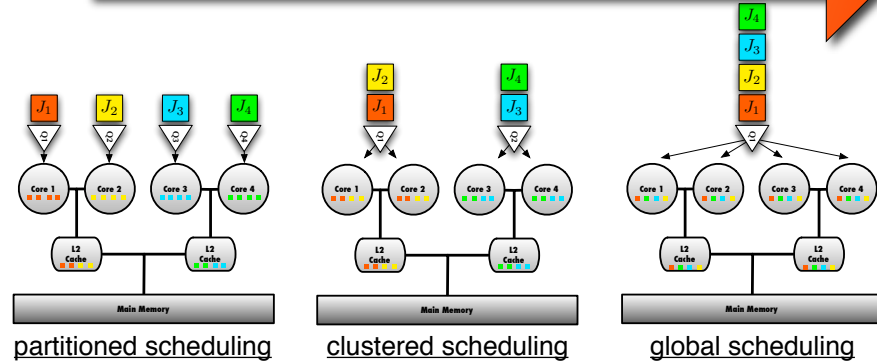




# Clustered Scheduling

smaller clusters = harder bin packing instance

larger clusters = higher overheads



# Semi-Partitioned Scheduling

another generalization partitioned scheduling

## Partition first

- **Assign each task statically** to a processor if possible
- Keep track which tasks could not be assigned (if any)
- Details vary according to specific **semi-partitioned** algorithm

## Split remaining tasks across multiple processors

- **Split** each **unassigned task** into multiple “portions” or “chunks”
- **Distribute** portions/chunks among multiple processors
  - E.g., split each job into **subjobs with precedence constraints**
  - Alternatively, do not migrate jobs, but **vary a task’s processor assignment** over time (soft real-time)

# Summary

## Approaches

- Partitioned
- **Global**
- Hybrid
  - Clustered
  - Semi-Partitioned

## Priorities

- Task-Level Fixed Priority
- Job-Level Fixed Priority
- **Job-Level Dynamic Priority**

## Optimal Online Scheduling

- **Implicit deadlines:** requires global job-level dynamic priority scheduler
- Constrained deadlines: does not exist
- Arbitrary deadlines: does not exist