



Prosa: A Case for Readable Mechanized Schedulability Analysis

Felipe Cerqueira, Felix Stutz, Björn B. Brandenburg



Max
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Institute
for
Software Systems

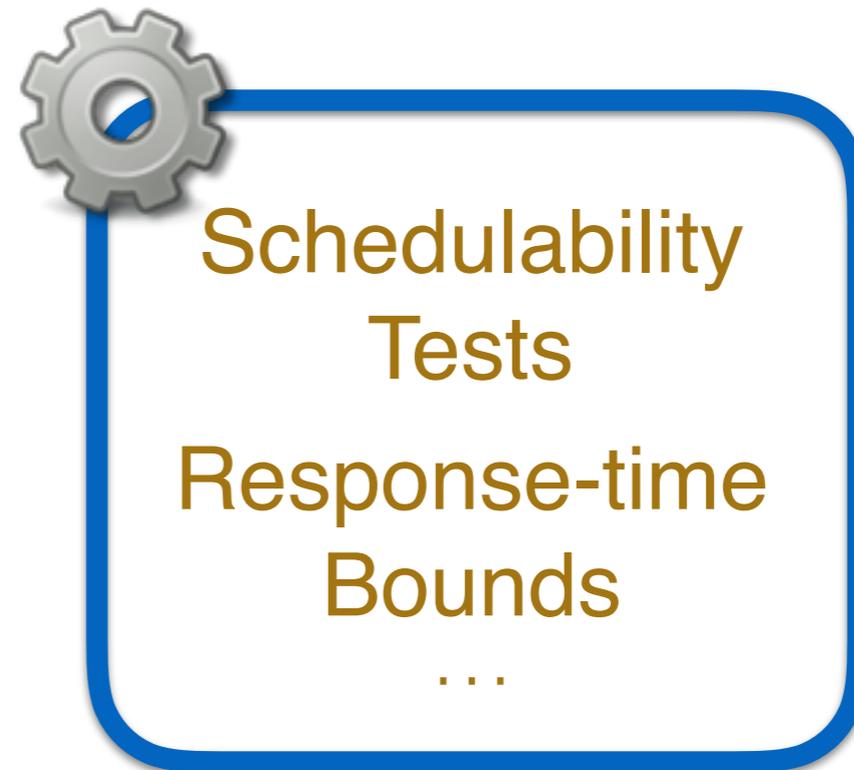


Prosa

Open-source foundation for formally
proven schedulability **analysis**



**Readable Formal
Specification**



**Mechanized
Proofs**

This Talk

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Mechanized proofs provide an opportunity to avoid the **correctness pitfalls in real-time scheduling**.

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By focusing on readability and by maintaining the established proof culture, mechanized proofs **can reach the community at large**.

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By focusing on readability and by maintaining the established proof culture, mechanized proofs **can reach the community at large**.

Thanks to mature proof assistants and libraries, Prosa allows mechanizing **recent and complex** schedulability analyses in **reasonable time**.

Outline of the Talk

Why mechanized proofs?

Challenges & Principles

A Taste of Prosa

Outline of the Talk

Why mechanized proofs?

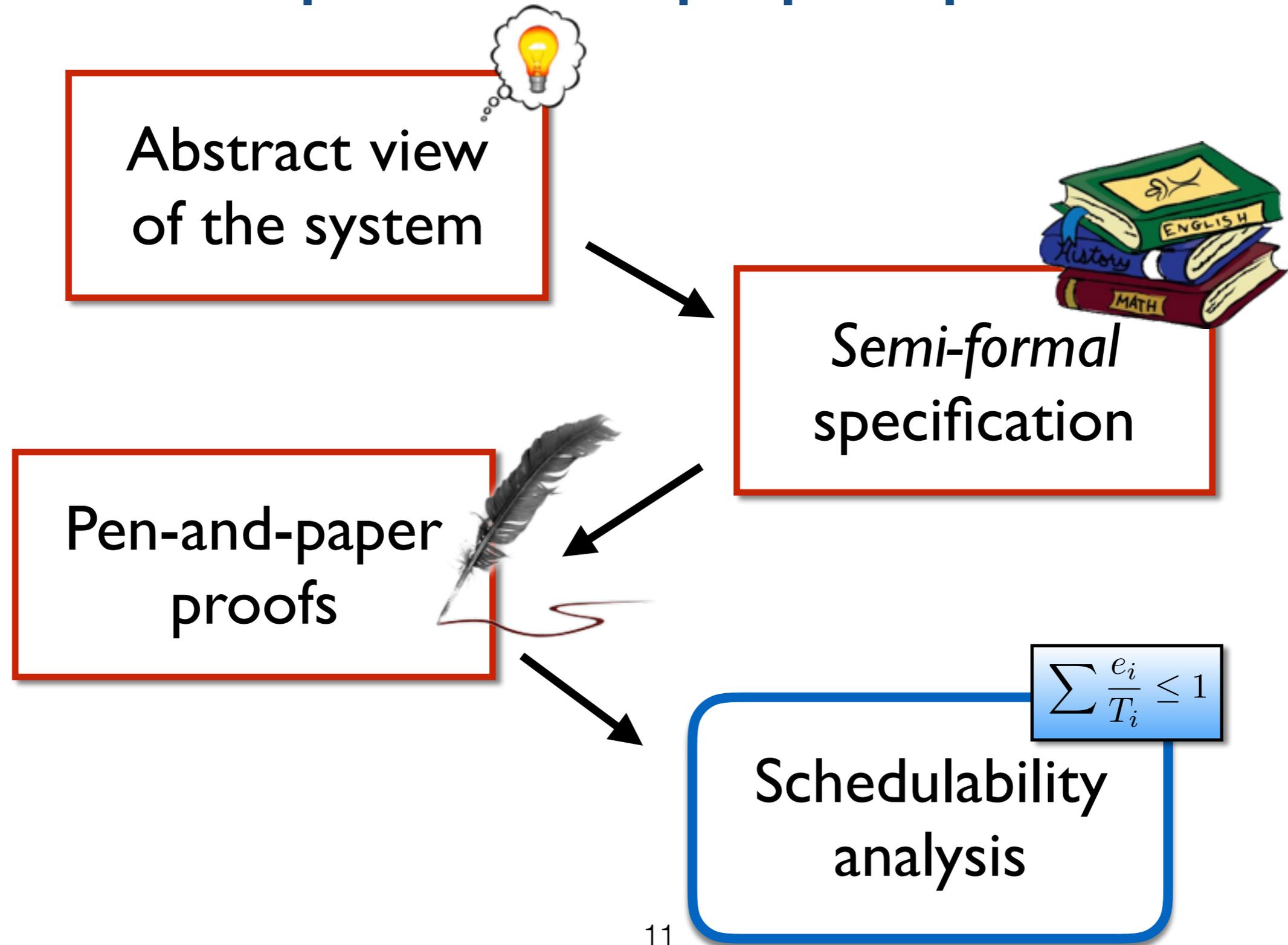
Challenges & Principles

A Taste of Prosa

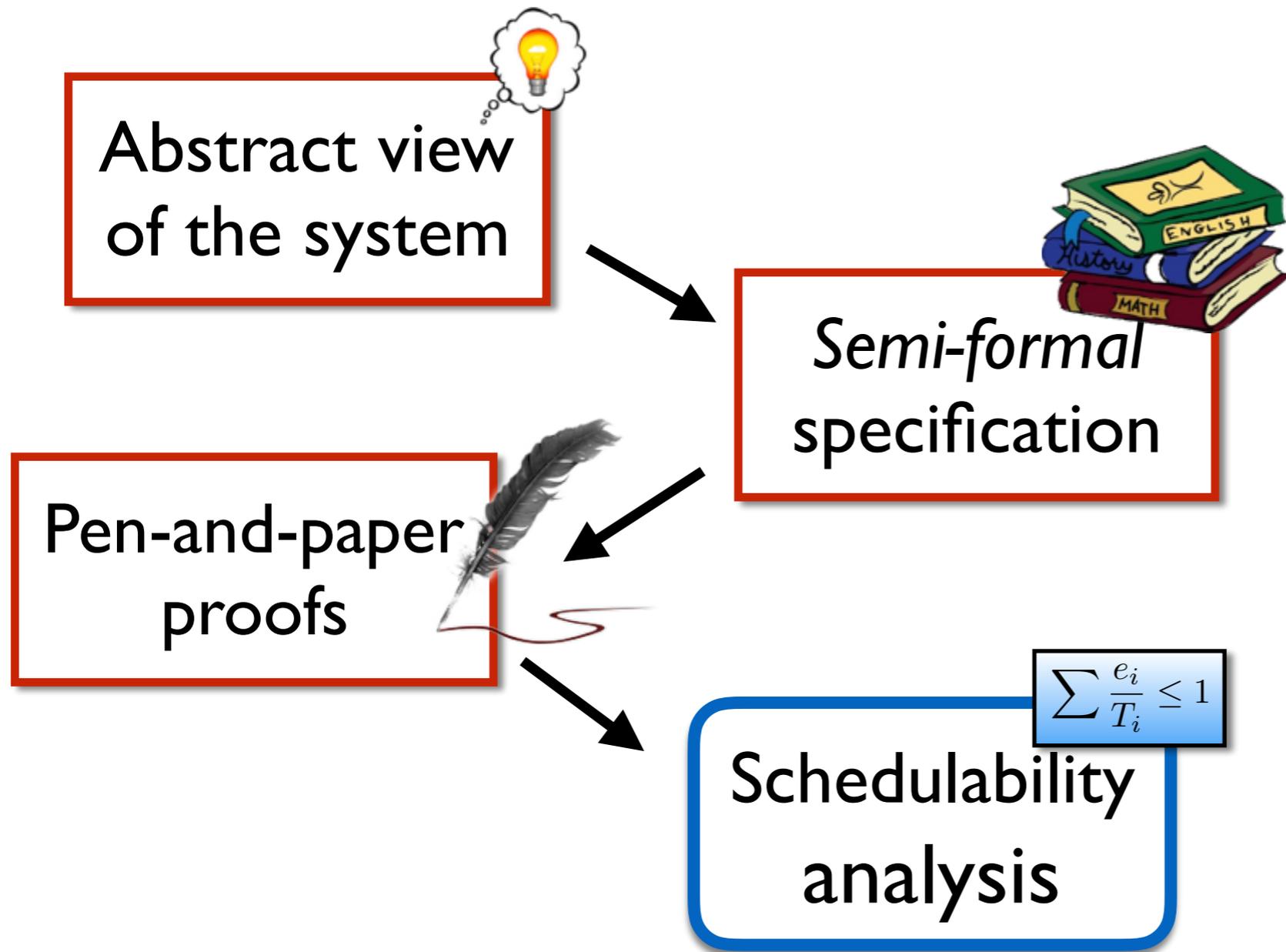
What do we mean
by **mechanized**?

RTS theory has been built
with pen-and-paper proofs

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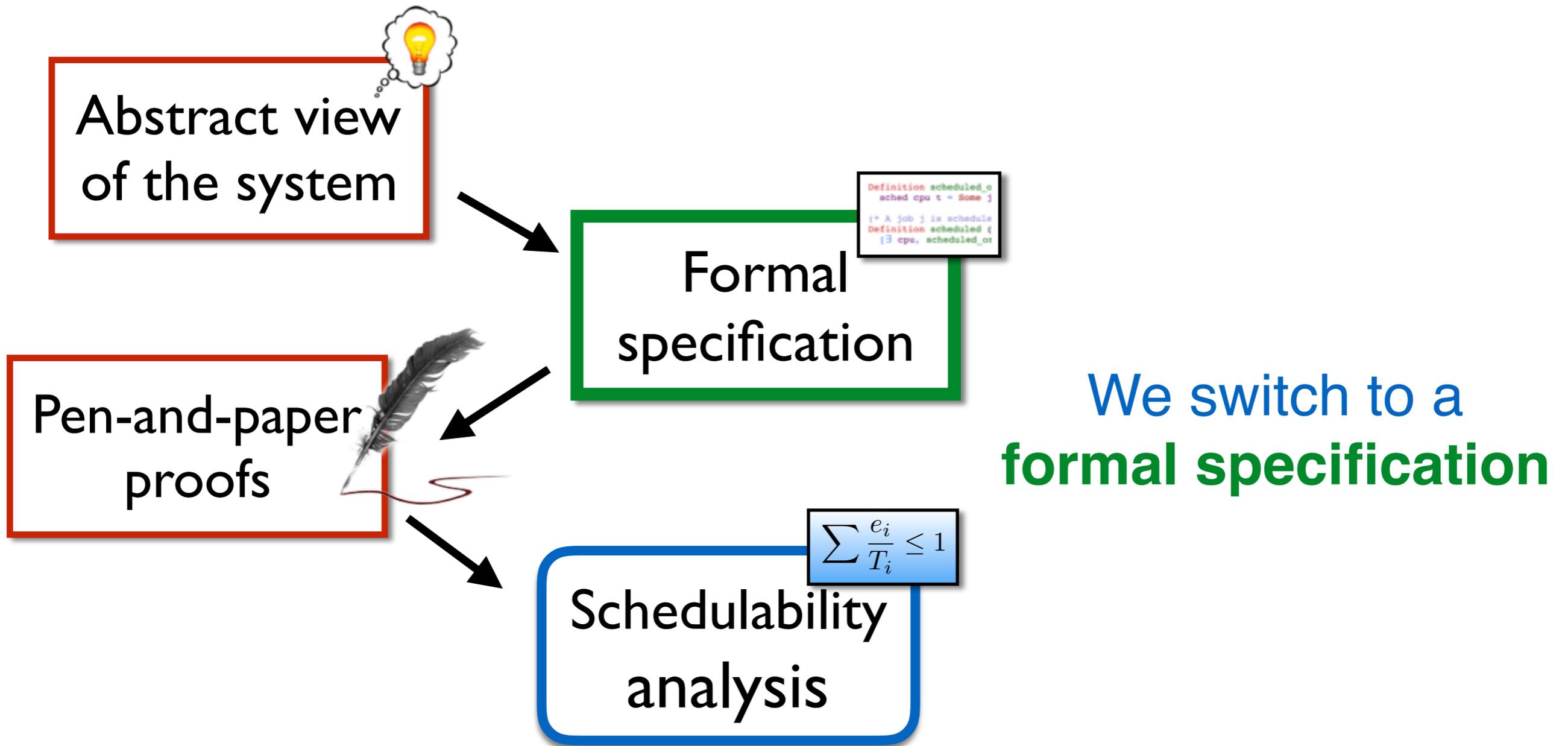


What is mechanized schedulability analysis?

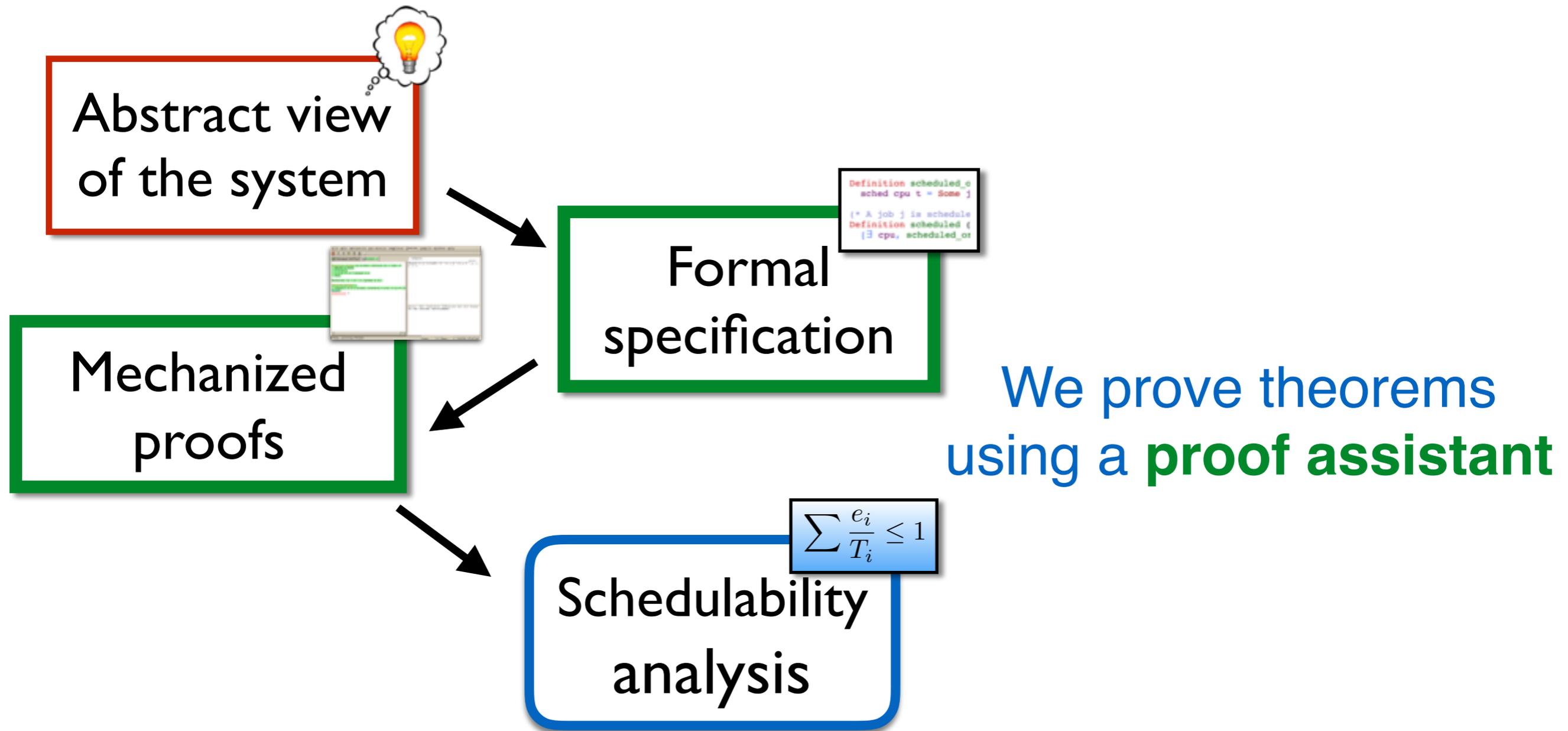


What is the difference?

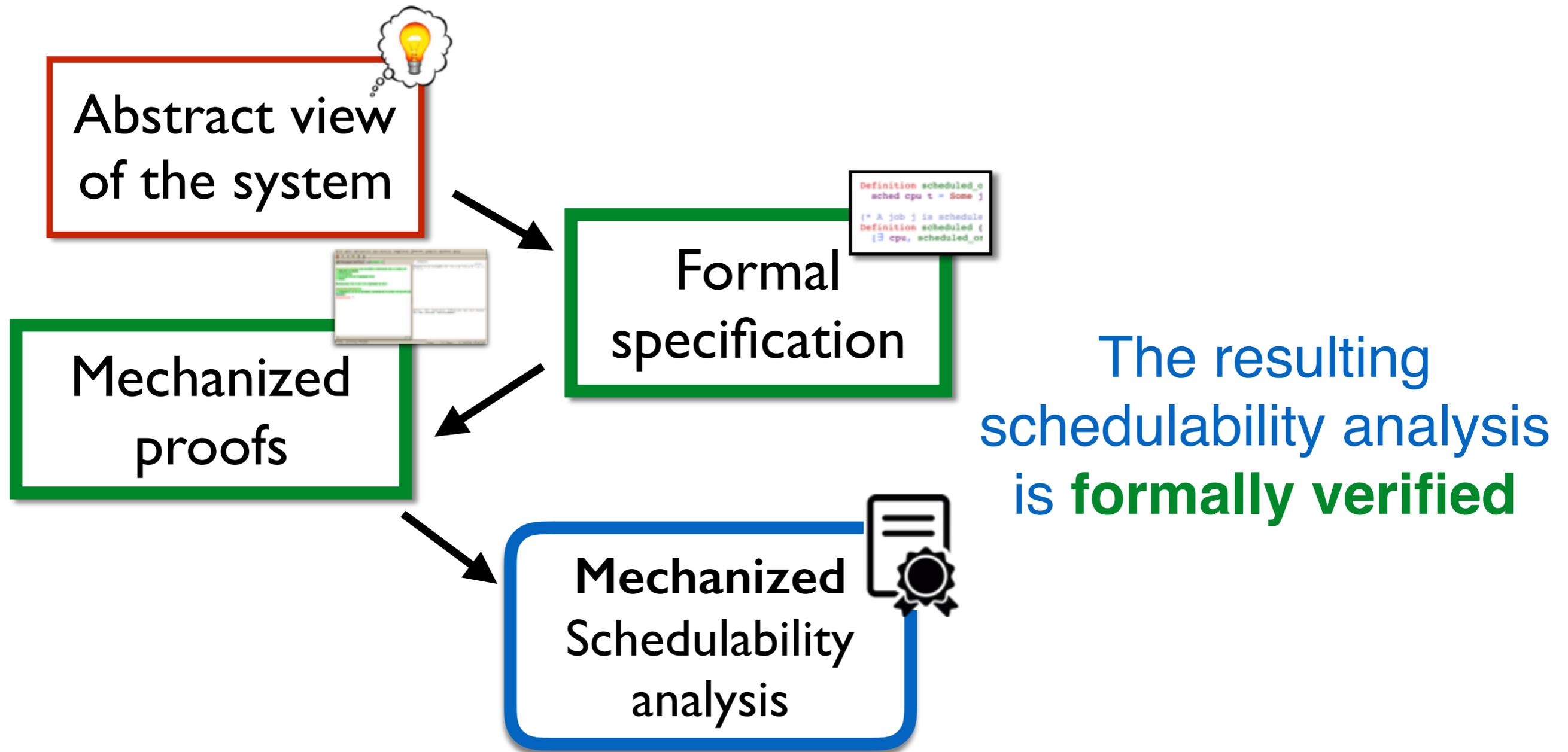
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Why mechanized proofs?

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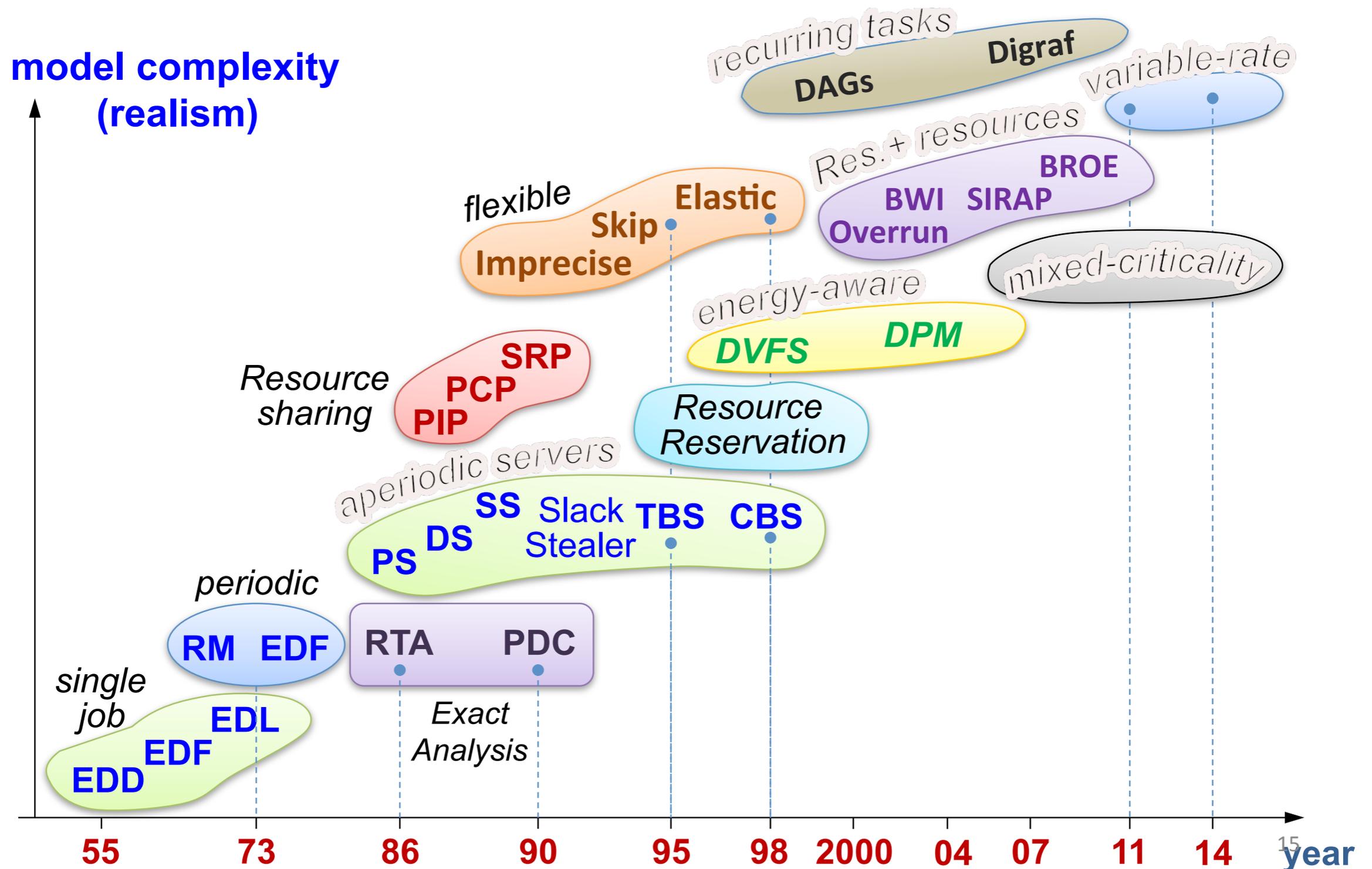
Guaranteed Correctness

```
graph TD; A[Guaranteed Correctness] --> B[Trustworthy Extensions]; A --> C[Safe Composition]
```

**Trustworthy
Extensions**

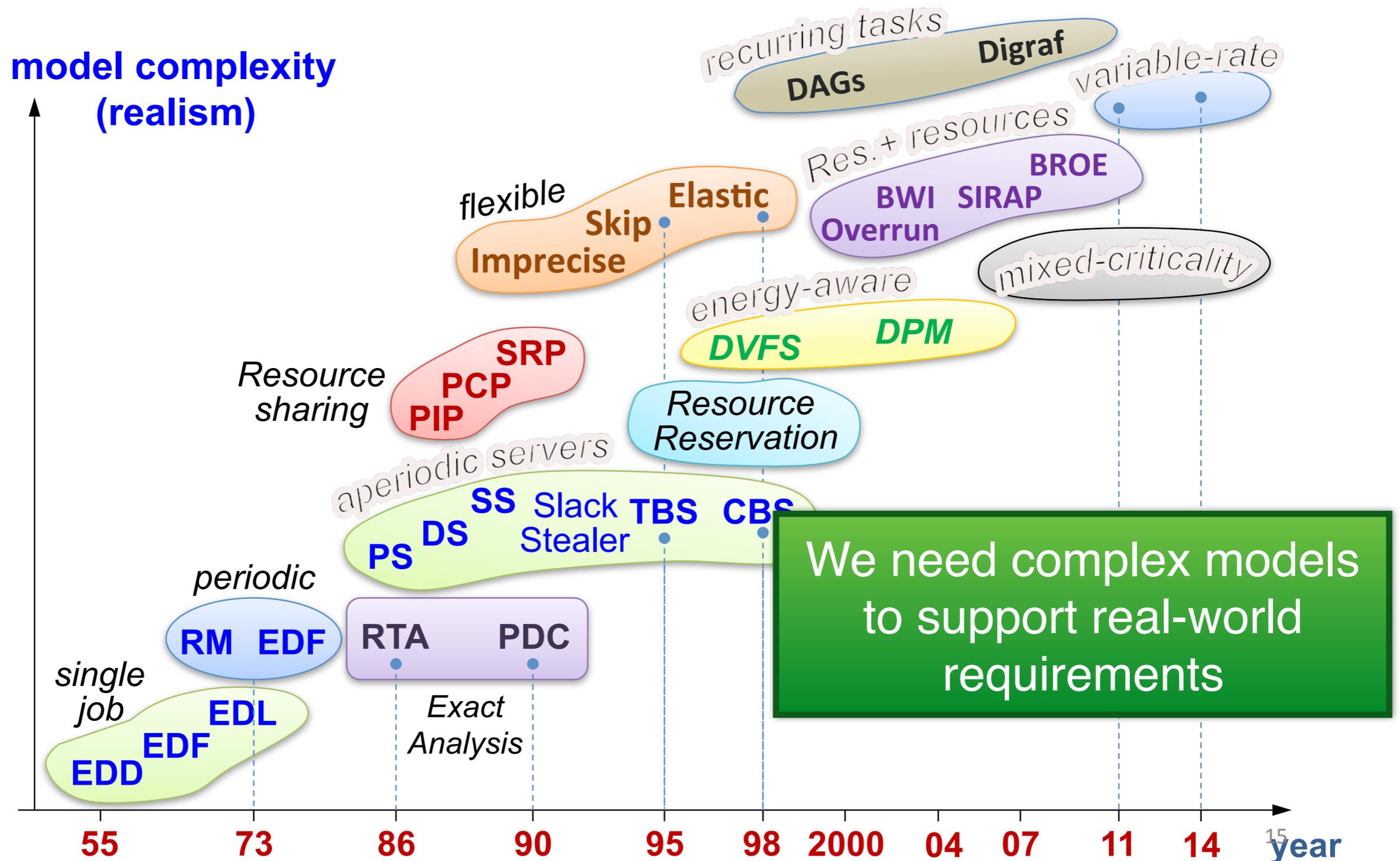
**Safe
Composition**

RTS have become more complex



Source: G. Buttazzo (Keynote @ RTSS'14)

RTS have become more complex



Source: G. Buttazzo (Keynote @ RTSS'14)

This complexity comes with a price

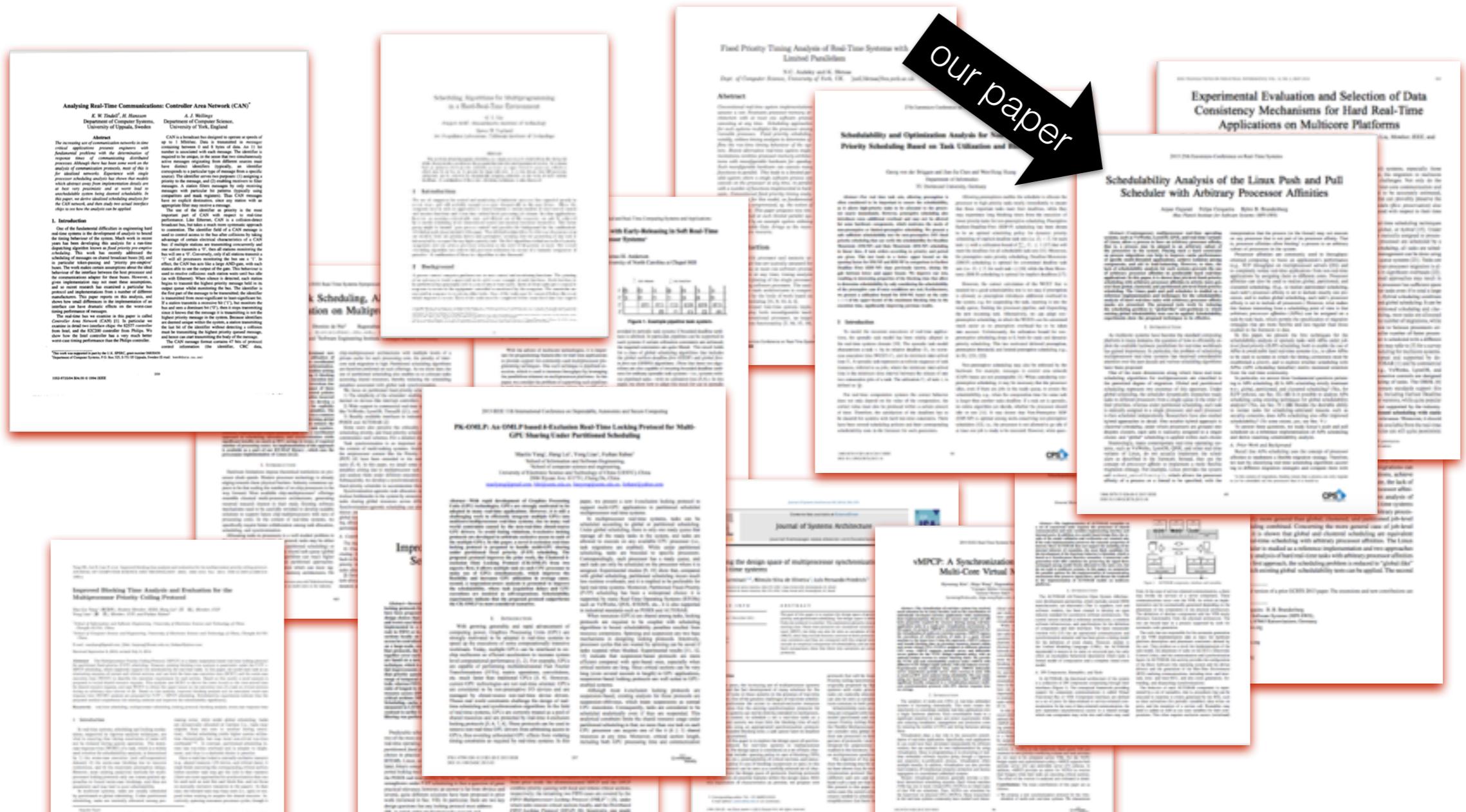
The original analysis for CAN **had a bug** that remained undetected from 1994 to 2006 [1].



[1] Davis, R. I., Burns, A., Bril, R. J., & Lukkien, J. J. "Controller Area Network (CAN) schedulability analysis: Refuted, revisited and revised." *Real-Time Systems*, 35(3), 239-272, 2007.

Bugs are no longer an exception

Proofs have become so complicated that they often contain bugs.



Analysis for safety-critical systems?



Analyzing Real-Time Communications: Controller Area Network (CAN)

Scheduling Algorithms for Multitasking in a Real-Time Environment

Fixed Priority Scheduling Analysis of Real-Time Systems with Limited Periodicity

Y.-T. Lee and S. Moon
Dept. of Computer Science, University of Park, U.S.

Abstract: This paper presents a scheduling analysis for real-time systems with limited periodicity. The analysis is based on the concept of a virtual deadline and a virtual period. The analysis is applied to a real-time system with limited periodicity.

Experimental Evaluation and Selection of Data Consistency Mechanisms for Hard Real-Time

How to ensure that schedulability analysis is actually correct?



Analysis for safety-critical systems?



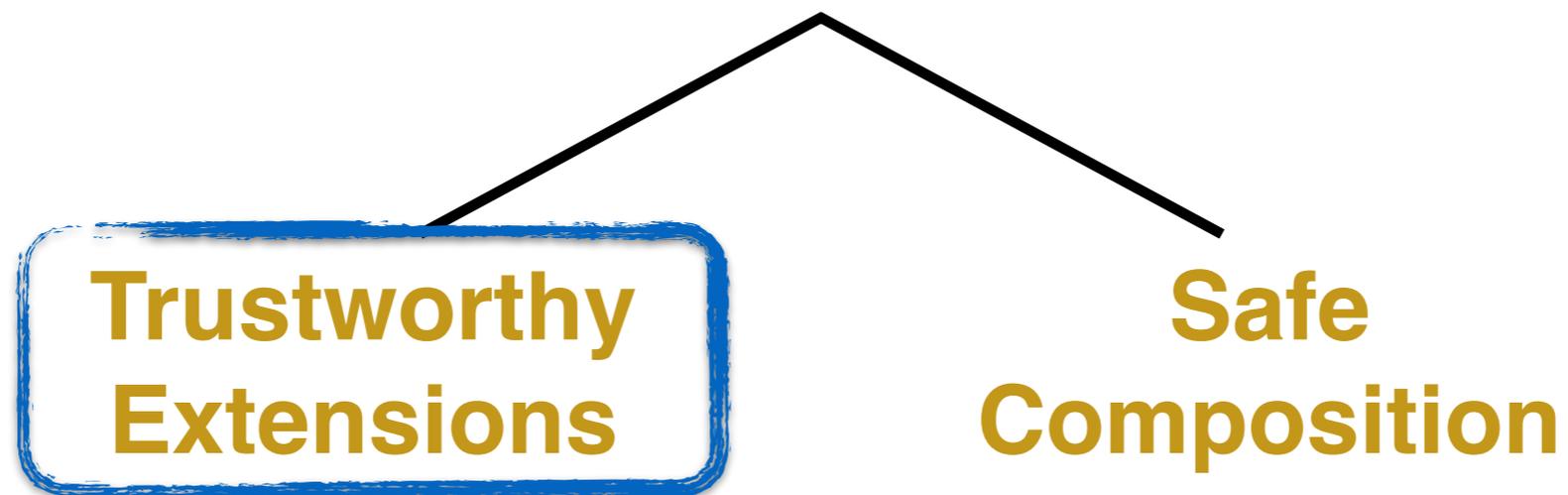
How to ensure that schedulability analysis is actually correct?

Mechanized proofs

Opportunity: correctness is **inherently guaranteed.**

Why mechanized proofs?

Guaranteed Correctness

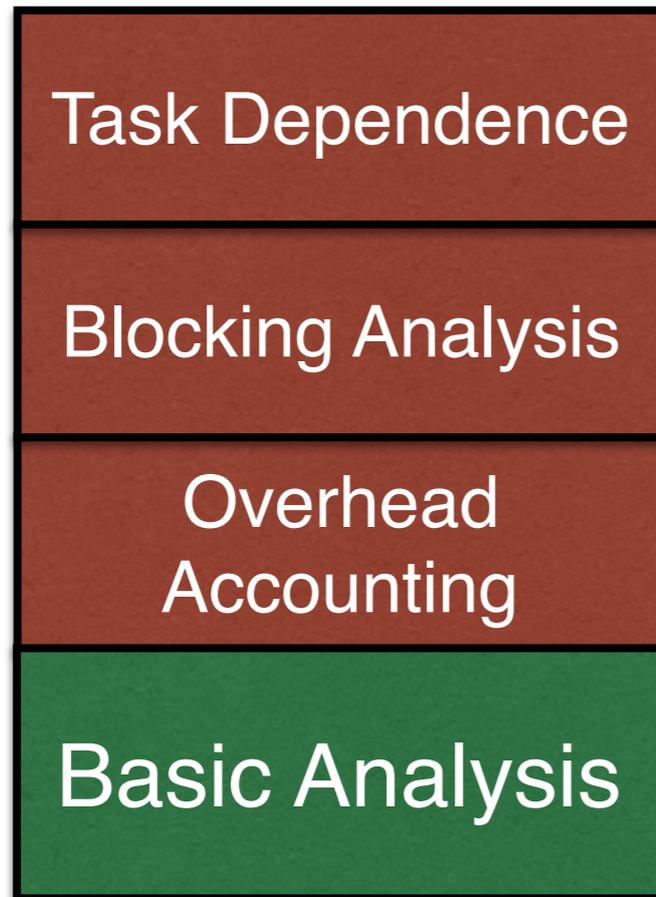


Analyses sometimes need refining

Basic Analysis

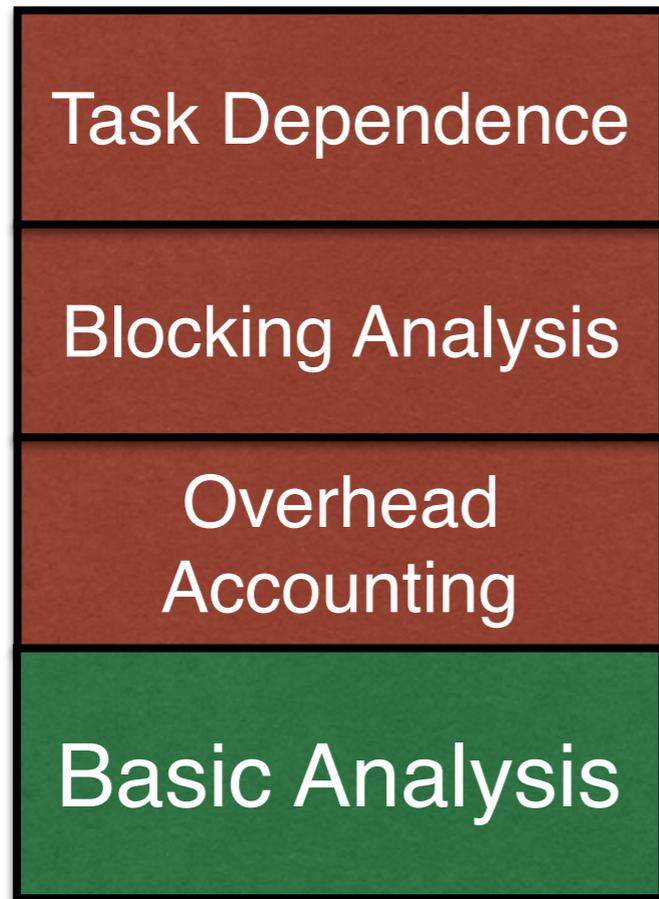
In most analyses, practical details are assumed to be negligible.

Analyses sometimes need refining



But when deploying actual systems,
we might need to **refine the analysis**.

Analyses sometimes need refining



We call these extensions (i.e., same results + tweaks) **neighboring proofs.**

But when deploying actual systems, we might need to **refine the analysis.**

Example: incorporating release jitter

	Basic RTA	RTA with Jitter
Uniprocessor	$R_i \leftarrow e_i + \sum_{\tau_j \in hp_i} \left\lceil \frac{R_i}{T_j} \right\rceil e_j$	$r_i \leftarrow e_i + \sum_{\tau_j \in hp_i} \left\lceil \frac{r_i + J_j}{T_j} \right\rceil e_j$ $R_i = J_i + e_i + r_i$

It has been known for more than 20 years how to incorporate release jitter into uniprocessor RTA [3].

[3] Audsley, N., Burns A., Richardson, M., Tindell, K., and Wellings, A. "Applying new scheduling theory to static priority pre-emptive scheduling," Software Engineering Journal, vol. 8, no. 5, pp. 284-292, 1993.

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Multiprocessor	$R_i \leftarrow e_i + \frac{1}{m} \cdot \sum_{\tau_j \in hp_i} \left\lceil \frac{I_j(R_i)}{T_j} \right\rceil r_j$	<p style="text-align: center; color: red; font-weight: bold;">???</p>

But this result has not been proven for multiprocessor RTA.

Can we do the same for multiprocessors?

	Basic RTA	RTA with Jitter
Uniprocessor	$R_i \leftarrow e_i + \sum_{\tau_j \in hp_i} \left\lceil \frac{R_i}{T_j} \right\rceil e_j$	$r_i \leftarrow e_i + \sum_{\tau_j \in hp_i} \left\lceil \frac{r_i + J_j}{T_j} \right\rceil e_j$ $R_i = J_i + e_i + r_i$
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Just sum up
the max jitter?

The answer is that **we don't know**

Different system models have **different** assumptions.
What if changing the model **breaks some existing proof?**

Recent case: self-suspending tasks

Misuse of release jitter in the analysis caused bugs in 12 papers related to self-suspensions!

Excerpt from [1]:

- **Incorrect quantification of jitter** for dynamic self-suspending task systems, which was used in [3,4,37,58]. This misconception was unfortunately adopted in [12, 14, 28, 36, 40, 73, 74, 76] to analyze the worst-case response time for partitioned multiprocessor real-time locking protocols.

How to derive safe extensions?

How to derive safe extensions?



Mechanized
proofs

Opportunity: neighboring proofs are conducted **systematically**.

We just need to **refine the analysis** and
let the proof assistant recheck the proofs.

If there is a bug, { **it will always be detected.**
we know exactly what to fix.

Why mechanized proofs?

Guaranteed Correctness

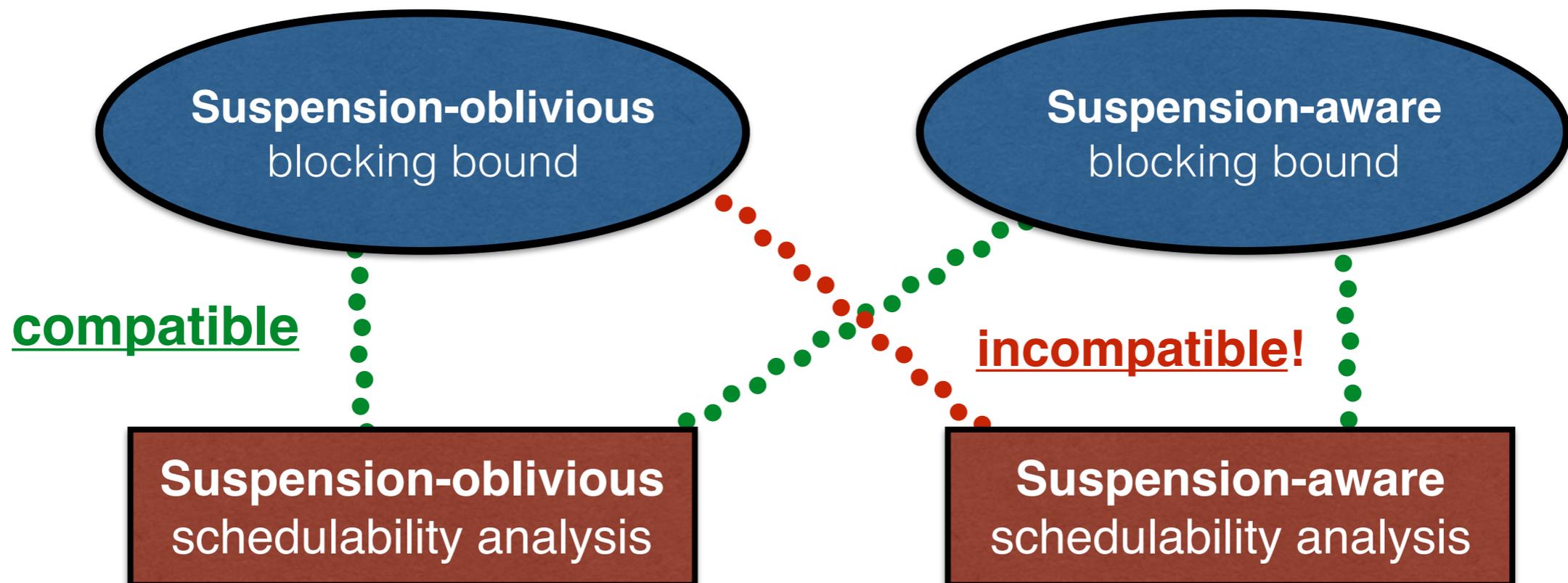
**Trustworthy
Extensions**

**Safe
Composition**

Sometimes we have to combine different analyses

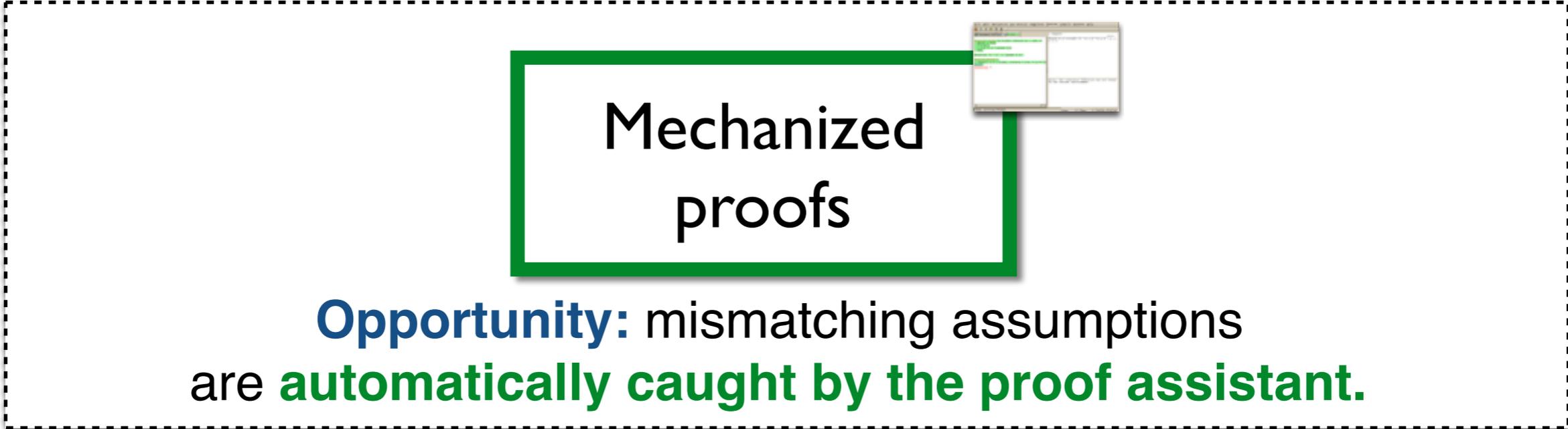
Even if each analysis is **individually correct**,
they should not be combined if assumptions mismatch.

Example:



How to avoid mismatching assumptions?

How to avoid mismatching assumptions?



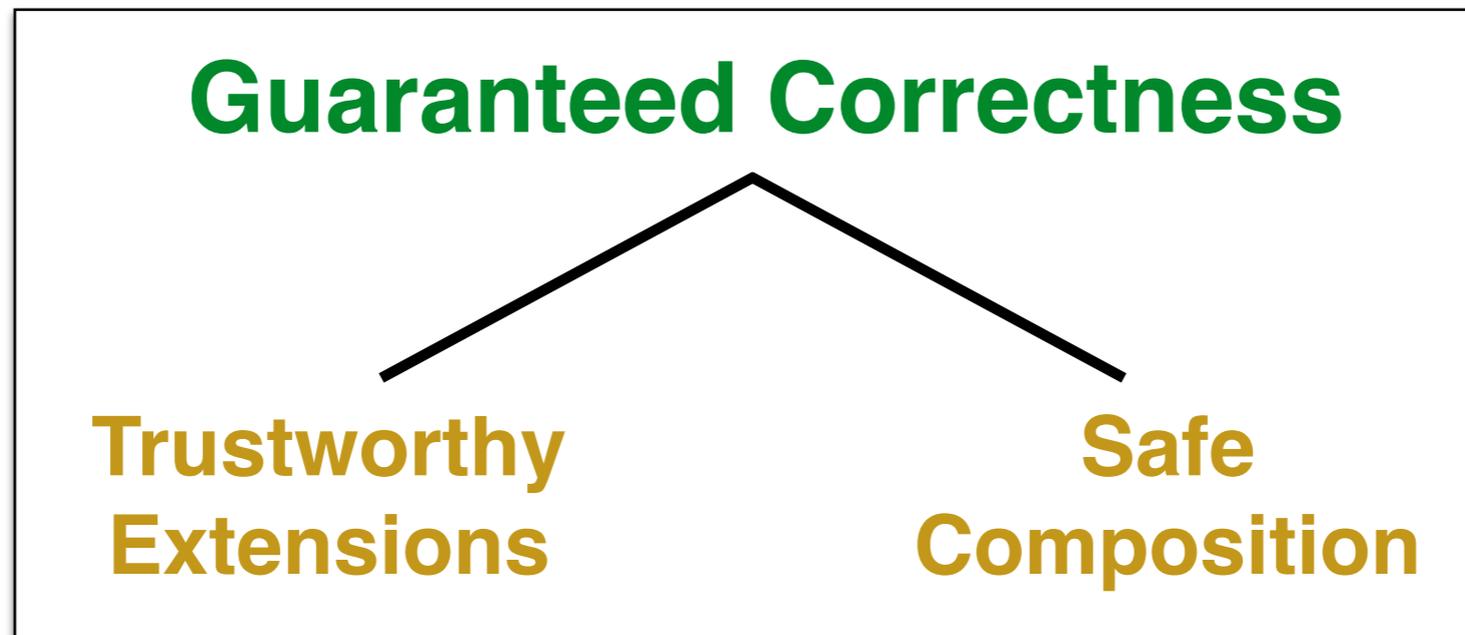
Mechanized
proofs

Opportunity: mismatching assumptions
are **automatically caught by the proof assistant.**

We just need to **avoid stating contradictory assumptions.**

But this can also be mechanically verified!

No more correctness pitfalls



Mechanized proofs provide an opportunity to avoid the **correctness pitfalls in real-time scheduling**.

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Challenges & Principles

A Taste of Prosa

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Verification has many challenges

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Verification has many challenges

“Formal specifications are complex and full of symbols.”

“It might take many decades to verify all we know about real-time scheduling.”

“Knowledge about formal methods tends to be restricted to few research groups.”

But there's an opportunity to change...

Principles & Goals of Prosa

1. **Readability** is crucial
2. Some proofs are **more important than others**
3. We should **maintain the proof culture**
4. **Community involvement**

Principle 1: Readability is crucial

The specification should be accessible to researchers with no previous experience with formal methods.

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The specification should be accessible to researchers with no previous experience with formal methods.

We favor:

Many lemmas, short proofs (few dozen lines)

Long, verbose names and few cryptic symbols

Heavy use of documentation

Complex notation harms readability

Duration Calculus [Yuhua and Chaochen, 1994]

Furthermore, if there exists a ready task which is not running, then no processor should be idle. So let,

$$SCH_m \hat{=} \left(\begin{array}{l} \neg \diamond \left(\begin{array}{l} true; Run(S) \\ \wedge (\bigvee_{i \in S} \bigvee_{j \in \alpha - S} (Urgt(j, i) ; \llbracket p_j.rdy \rrbracket \wedge Run(S))) \end{array} \right) \\ \wedge \square (Run(S) \wedge \llbracket \bigvee_{i \in \alpha - S} p_i.rdy \rrbracket \Rightarrow \#S = m) \end{array} \right)$$

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$$SCH_m \hat{=} \left(\begin{array}{l} \neg \diamond \left(\begin{array}{l} true; Run(S) \\ \wedge (\forall i \in S \forall j \in \alpha - S (Urgt(j, i) ; [p_j.rdy] \wedge Run(S))) \end{array} \right) \\ \wedge \square (Run(S) \wedge [\forall i \in \alpha - S p_i.rdy] \Rightarrow \#S = m) \end{array} \right)$$

Prosa

(* A scheduler is work-conserving iff all processors are busy (non-idle) whenever a job is backlogged. *)

Definition `work_conserving` :=

`∀ j ∀ t,`

`backlogged job_cost sched j t →`

`∀ cpu, ∃ j_other,`

`scheduled_on sched j_other cpu t.`

Long names and few symbols

```
Definition work_conserving :=  
  ∀ j ∀ t,  
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Definition work conserving :=

$\forall j \forall t,$
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A scheduler is work-conserving iff...

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A scheduler is work-conserving iff...

...for every job j and time t ...

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A scheduler is work-conserving iff...

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A scheduler is work-conserving iff...

...for every job j and time t ...

...if job j is backlogged at time t , ...

...then every processor cpu has a job j_other ...

...that is scheduled on cpu at time t .

Principle 2: Some proofs are more important than others

To make progress, we should focus on practical results.

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To make progress, we should focus on practical results.

We should formalize **recent analyses** and move towards **multiprocessor scheduling**.

Critical results should be proven first.
E.g., proving **analysis safety** is more important than **termination, time complexity or optimality**.

Principle 3: Maintain the proof culture

To ensure accessibility, we should reuse the established proof style of the real-time systems community.

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To ensure accessibility, we should reuse the established proof style of the real-time systems community.

We avoid **complex logics** (e.g., temporal operators) and **advanced constructs** from the proof assistant (e.g., records, canonical structures, etc.).

We favor instead **first-order logic, lists, functions, basic arithmetic.**

Unusual notation discourages adoption

EDF Optimality in PPTL [Zhang, 2014]

Lemma 6. *If P_i overflows at $t = kT_i$, there is no idle time unit in $[(k - 1)T_i, kT_i]$.*

That is,

$$\text{Sch} \supset \left(\bigcirc^{kT_i} (ac_i < C_i) \rightarrow \bigcirc^t \left(\bigvee_{j=1}^m r_j = 1 \right) \right) \quad (k - 1)T_i < t < kT_i.$$

Prosa — Definition of Instantaneous Service

```
Definition service_at (t: time) :=  
  \sum_(cpu < num_cpus | scheduled_on j cpu t) 1.
```

LaTeX-like operators improve readability

Instantaneous Service

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Sum over each processor...

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Instantaneous Service

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Sum over each processor...

...where job j is scheduled...



LaTeX-like operators improve readability

Instantaneous Service

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Definition service_at (t: time) :=  
  \sum_(cpu < num_cpus | scheduled_on j cpu t) 1.
```

Sum over each processor...

...where job j is scheduled...

...of 1 (i.e., a count).

Principle 4: Community involvement

Vision: shared repository of real-time scheduling concepts and proofs.

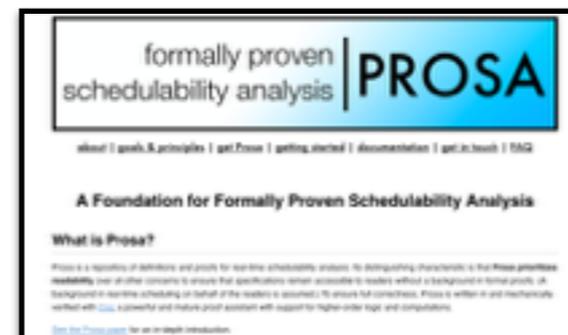
Principle 4: Community involvement

Vision: shared repository of real-time scheduling concepts and proofs.

We encourage **participation by the community:**



Check out our website:
prosa.mpi-sws.org



Mechanized proofs can reach the community at large

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By focusing on readability and by maintaining the established proof culture, mechanized proofs **can reach the community at large.**

Outline of the Talk

Why mechanized proofs?

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A Taste of Prosa

Prosa is a collection of definitions,
assumptions and theorems

Definitions

Assumptions

Theorems

Prosa covers many concepts from real-time scheduling

Definitions

Library schedule: instantaneous service,
cumulative service,
job is pending, job is complete..

Library interference: total interference,
per-task interference..

Library platform: work conservation,
priority enforcement..

Assumptions

Theorems

Assumptions can be easily checked (~10–15 in each analysis)

Definitions

Assumptions

Hypothesis H_completed_jobs_dont_execute:
completed_jobs_dont_execute job_cost sched.

Hypothesis H_enforces_FP_policy:
enforces_FP_policy job_cost job_task sched higher_priority.

Hypothesis H_work_conserving: work_conserving job_cost sched.

Hypothesis H_sequential_jobs: sequential_jobs sched.

[...]

Theorems

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Definitions

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Hypothesis `H_completed_jobs_dont_execute:`
`completed_jobs_dont_execute` `job_cost` `sched.`

In any given schedule and for any
given actual job execution costs, ...

Theorems

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Definitions

Assumptions

Hypothesis `H_completed_jobs_dont_execute:`
`completed_jobs_dont_execute job_cost sched.`

In any given schedule and for any
given actual job execution costs, ...

...jobs do not execute after completion.

Theorems

Assumptions can be easily checked (~10–15 in each analysis)

Definitions

Assumptions

Hypothesis `H_completed_jobs_dont_execute`:
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Definition `completed_jobs_dont_execute` :=
 $\forall j \forall t,$
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Theorems

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For every job j at any time t ,

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`completed_jobs_dont_execute job_cost sched.`

Definition `completed_jobs_dont_execute` :=
 $\forall j \forall t,$
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For every job j at any time t ,

...the service received by j is no larger than its cost.

Theorems

Theorems are proven in small steps using **lemmas**

Definitions

Assumptions

Theorems

Theorem `workload_bounded_by_W` :
`workload_of tsk t1 (t1 + delta) ≤ workload_bound.`

Theorems are proven in small steps using **lemmas**

Definitions

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We upper-bound the workload of a task...

Theorem workload_bounded_by_W :
workload_of_tsk t1 (t1 + delta) ≤ workload_bound.



Theorems are proven in small steps using lemmas

Definitions

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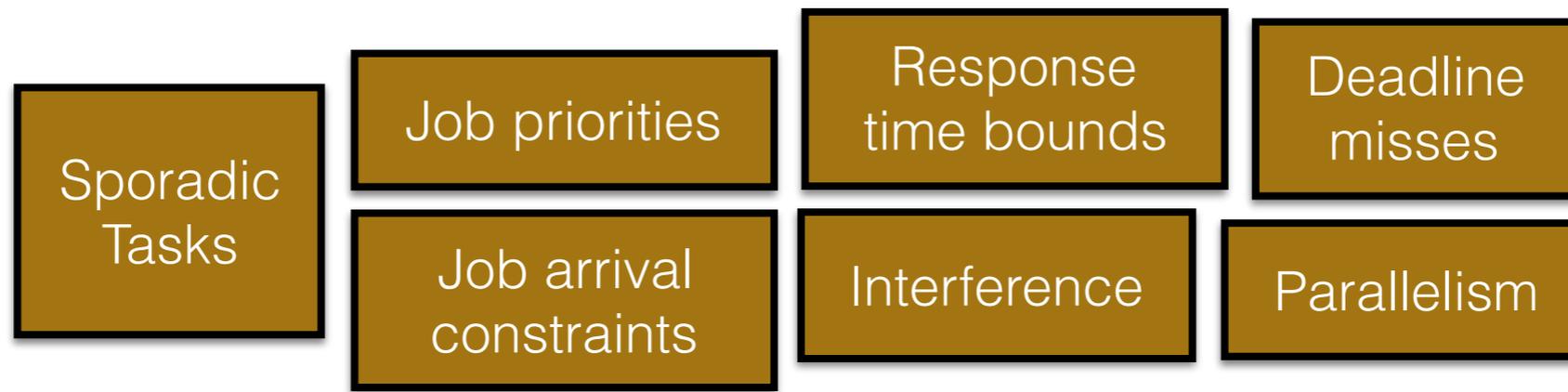
Lemma `workload_bound_many_periods_in_between` :
`job_arrival j_lst - job_arrival j_fst t` \geq `num_mid_jobs.+1`
 \times `(task_period tsk)`.

We upper-bound the workload of a task...

...based on the minimum distance between its first and last jobs in the interval.

Theorem `workload_bounded_by_W` :
`workload_of tsk t1 (t1 + delta)` \leq `workload_bound`.

Prosa covers many concepts and is well-documented



	Definition/Let	Lemma/Theorem
Total	714	699

We use short, easy-to-understand definitions.

	Specification	Proofs	Comments
Lines	6661	17104	3442

1 comment for every 2 lines of spec!

What we have proven so far

(in ~8 person months)

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- **Sporadic Task Model**

- [Workload-based interference bounds](#) for work-conserving and EDF schedulers

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- Definition and proofs of correctness and termination of **Bertogna and Cirinei's RTA for FP scheduling**
 - ➔ Same for **Bertogna and Cirinei's RTA for EDF scheduling**

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- **Sporadic Task Model**

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- Definition and proofs of correctness and termination of **Bertogna and Cirinei's RTA for FP scheduling**
 - ➔ Same for **Bertogna and Cirinei's RTA for EDF scheduling**
- **Implementation of a work-conserving scheduler** to test for contradictory assumptions

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- **Workload-based interference bounds** for work-conserving and EDF schedulers
- Definition and proofs of correctness and termination of **Bertogna and Cirinei's RTA for FP scheduling**
 - ➔ Same for **Bertogna and Cirinei's RTA for EDF scheduling**
- **Implementation of a work-conserving scheduler** to test for contradictory assumptions

- **Extensions**

- ➔ Same definitions and proofs for **workloads with release jitter**
 - ➔ Same definitions and proofs for **workloads with parallel jobs**
- } **novel results**

What we have proven so far

(in ~8 person months)

- Sporadic Task Model

- Workload-based interference bounds for work-conserving and EDF schedulers
- Definition and proofs of correctness and termination of Bertogna and Cirinei's

Thanks to mature proof assistants and libraries,
Prosa allows mechanizing **recent and complex**
schedulability analyses in **reasonable time.**

- Extensions

- ➔ Same definitions and proofs for workloads with release jitter
- ➔ Same definitions and proofs for workloads with parallel jobs

Future Work

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1. Correct **recently refuted proofs**

a) ~~APA scheduling~~ **(done! see prosa.mpi-sws.org/apa)**

b) Self-suspending tasks

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1. Correct **recently refuted proofs**
 - a) ~~APA scheduling~~ **(done! see prosa.mpi-sws.org/apa)**
 - b) Self-suspending tasks

2. Verify **practical results**
 - a) Semi-partitioned scheduling (e.g. C=D)
 - b) Blocking analysis
 - c) Overhead accounting

Future Work

1. Correct **recently refuted proofs**
 - a) ~~APA scheduling~~ **(done! see prosa.mpi-sws.org/apa)**
 - b) Self-suspending tasks

2. Verify **practical results**
 - a) Semi-partitioned scheduling (e.g. C=D)
 - b) Blocking analysis
 - c) Overhead accounting

3. Investigate how to integrate Prosa with **analysis tools** and **scheduler implementations**

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Pen-and-paper proofs are still useful.

We aim for readable specifications, but writing formal proofs remains non-trivial.

formally proven
schedulability analysis | **PROSA**

More info at prosa.mpi-sws.org

Mechanized proofs provide an opportunity to avoid the **correctness pitfalls in real-time scheduling**.

By focusing on readability and by maintaining the established proof culture, mechanized proofs **can reach the community at large**.

Thanks to mature proof assistants and libraries, Prosa allows mechanizing **recent and complex** schedulability analyses in **reasonable time**.

Backup slides

Generality of discrete time

Theorem 6 *A sporadic arbitrary-deadline task system \mathcal{T} is feasible with respect to continuous schedules iff it is feasible with respect to discrete schedules.*

Results about dense time could still be formalized with Coq libraries for real numbers, e.g. Coquelicot.

Working with Real Numbers

Coquelicot:

A User-Friendly Library of Real Analysis for Coq

Formalization of limits, continuity, differentiability,
Riemann integrals, series, etc.

More info at coquelicot.saclay.inria.fr

Library: Probability Theory

**Total/conditional probability, Bayes' theorem,
random variables and finite distributions**

```
Lemma prob_decomp: forall A B,  
  \Pr_d[A] = \Pr_d[A :&: B] + \Pr_d[A :&: ~:B].
```

Moreira, D. *Finite Probability Distributions in Coq (2012)*.

Related Work

Formalisms for schedulability analysis

Based on the Duration Calculus (DC) interval logic

- Proof of EDF optimality [*Yuhua and Chaochen 1994*]
— improved version [*Zhan 2000*]
- Schedulability condition of RM [*Schuzhen et al. 1999*]
- Simplified proofs and review [*Xu and Zhan 2008*]

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+ Formalism reduces ambiguity

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- + Formalism reduces ambiguity
- Complex logics and manual proofs
- Only uniprocessor scheduling

Earlier mechanized proofs

- Proof of **EDF optimality** using *Nqthm* [Wilding 1998]
- Analysis of the **Priority Ceiling** and **Priority Inheritance** Protocols [Zhang et al. 1999] [Dutertre 1999] [Dutertre and Stavridou 2000]
- Schedulability conditions based on **task phase** using *Coq* [De Rauglaudre 2012]
- Certified Computations of **Network Calculus** in Isabelle/HOL [Mabille et al. 2013]
- Implementation and proof of **EDF optimality** with Propositional Projection Temporal Logic (PPTL) in *Coq* [Zhang et al. 2014]

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+ Mechanically-checked

- No results about multiprocessors

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- Not widely adopted by our community

Model checking and timed automata

- Analysis of **uniprocessor** FP scheduling using UPPAAL
[Fersman et al. 2006]
- Analysis of **multiprocessor** FP and EDF scheduling of **periodic tasks** using UPPAAL and NuSMV
[Guan et al. 2007] [Guan et al. 2008] [Cordovilla et al. 2011]
- Analysis of **sporadic tasks** based on state exploration and automata reachability *[Baker and Cirinei 2007] [Geeraerts et al. 2012] [Burmyakov et al. 2015] [Sun and Lipari 2015]*

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+ Multiprocessor, exact schedulability analysis
- State explosion (≤ 10 tasks or 4 processors)

Avoiding contradictory assumptions

1. **Implement a scheduler function S** using the proof assistant (take pending jobs, sort by priority, assign to CPUs, ...).
2. **Prove that scheduler S satisfies every requirement** of the analysis (work-conserving, enforces priority, etc.) in an assumption-free context.
3. Since S is an actual algorithm, it is **impossible that two contradictory assumptions are satisfied by S .**